



Administrivia

- Enrollment
- Lottery
- Paper Reviews (Adobe)
- Data passwords
- My office hours



Good Cop Bad Cop

- Two or three ways to think about this class
 - Computer Science
 - Algorithms
 - Engineering
 - Brain Computer Interfaces
 - Neuroscience
 - Brain function



The CS Take Home

- Decoding Algorithms
 - Population Vector
 - Linear filtering
 - Particle/Kalman filtering
 - Neural Network
- Depths of Understanding
 - Implementation
 - Understanding



Where we Left Off

- Neural Coding Models?
 1. each neuron codes a particular value “the grandma cell”
 2. computer-like model where neuron firing patterns are like binary codes

- Problems?



The Answer?

- No one knows.

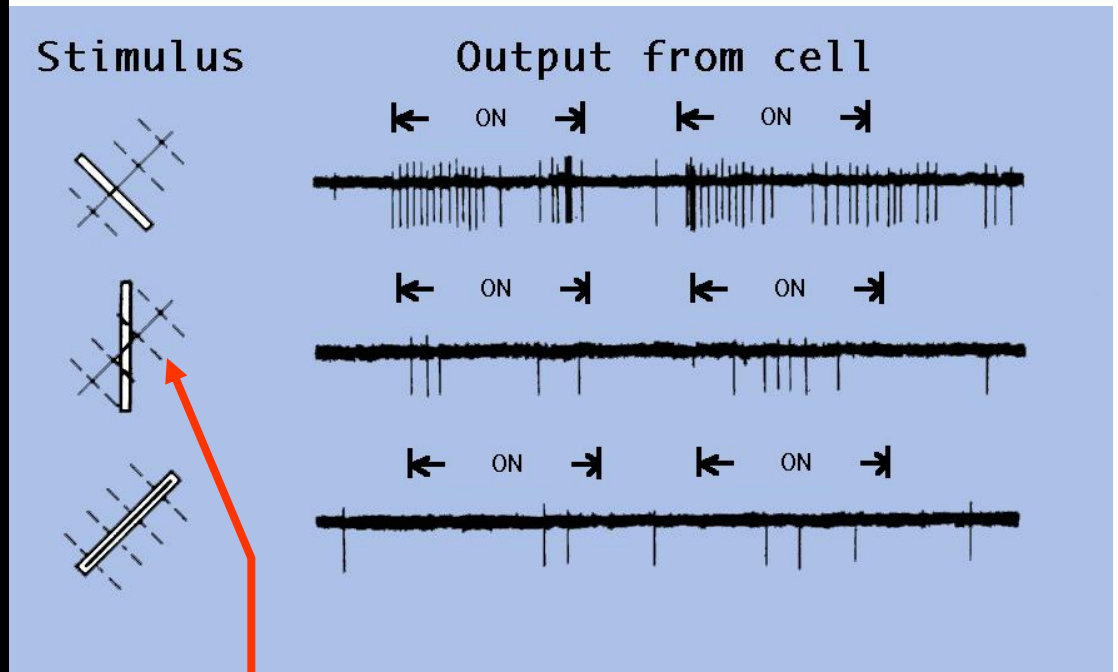
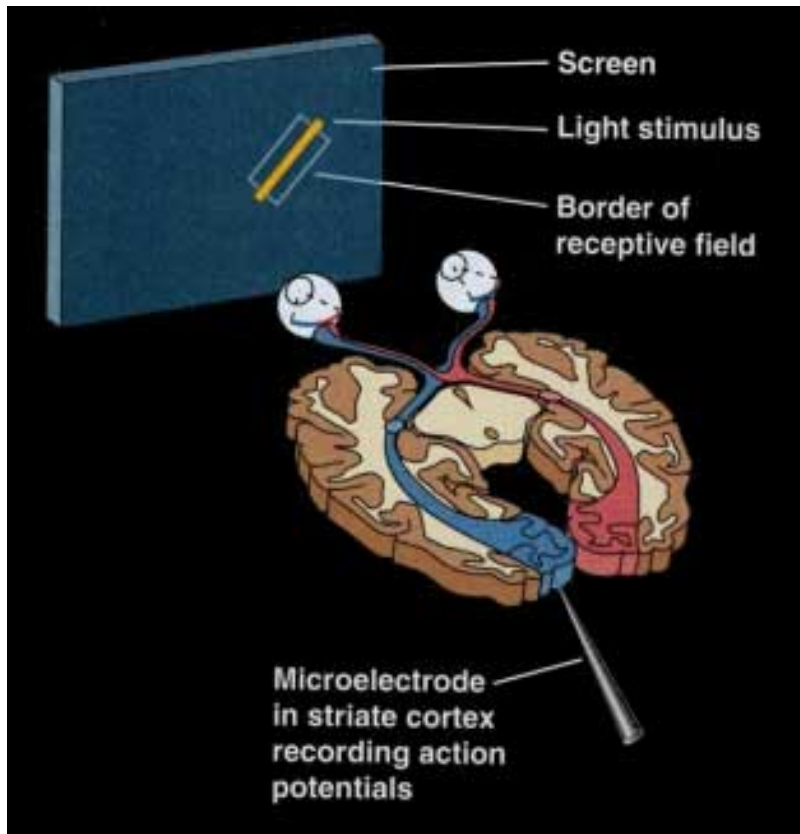


Tuned Response Everywhere

- Somatic sensory system
 - Receptive fields
- Visual Cortex
 - Receptive fields
- etc.



A Visual Cortex Example

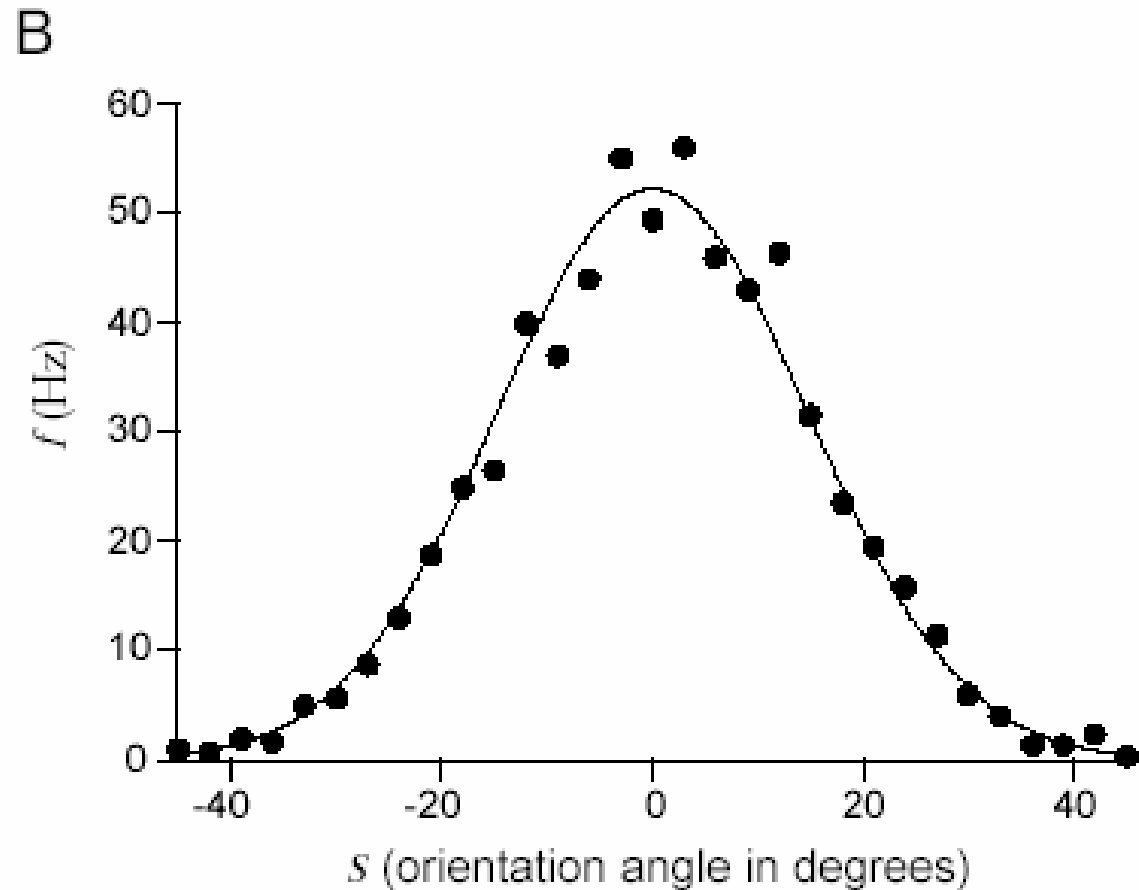
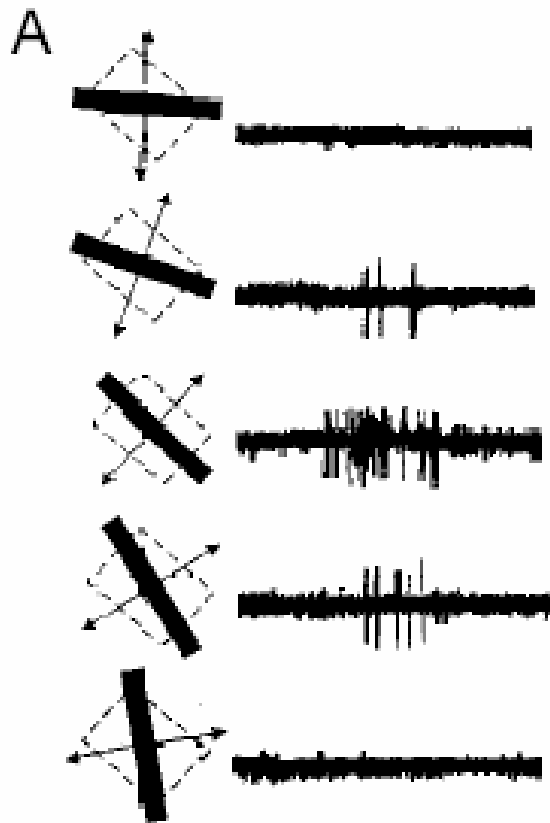


Hubel & Weisel, 1962

Orientation
Tuning



Rate Modulated by Orientation



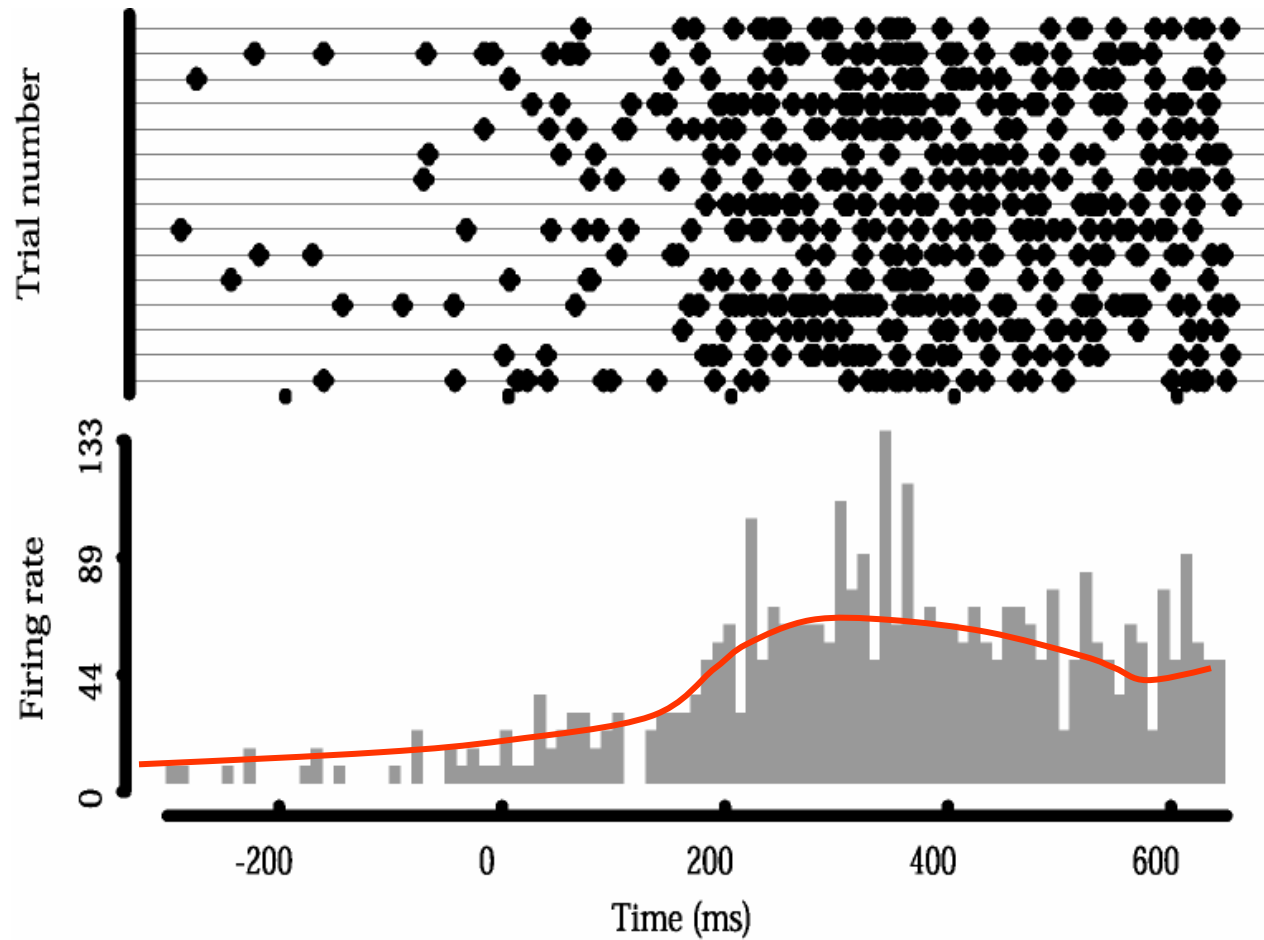


Rate Coding?

- Firing rate modulated by receptive field.
- But how is rate estimated/integrated?
- How can this represent multiple values robustly?



Estimating Firing Rate

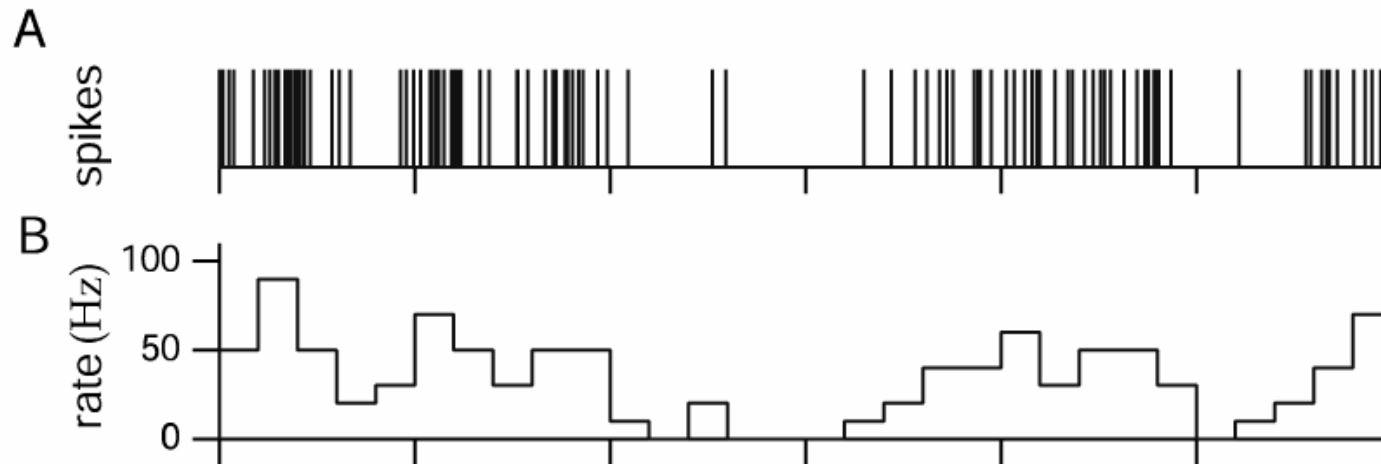


"rasters"

Source: Rob Kass



Estimating Firing Rate



Source: Zemel & McNaughton, NIPS2000 tutorial

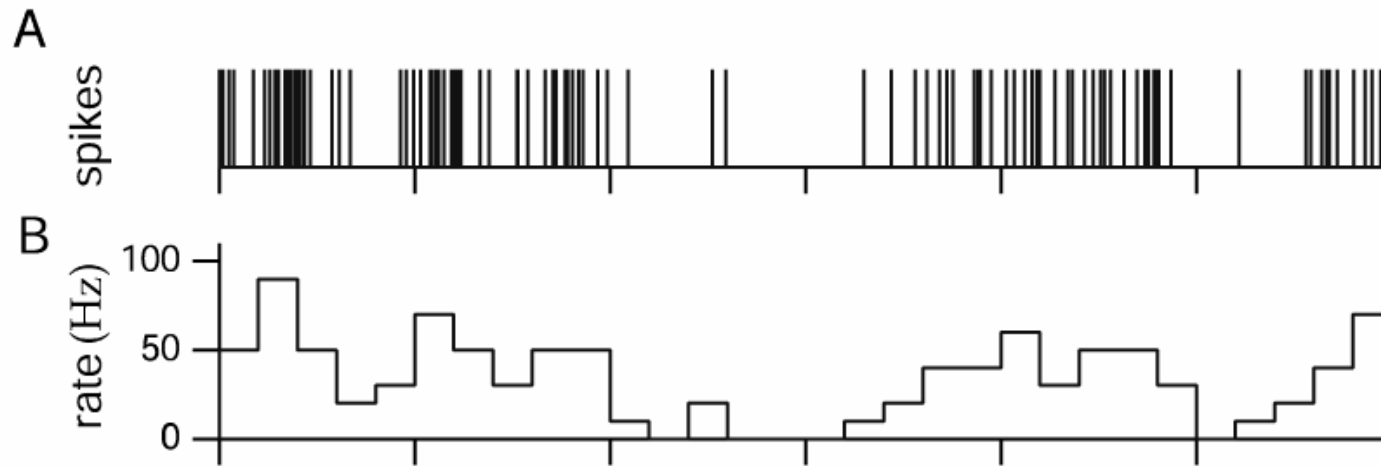
$$\text{rate} = (\# \text{ of spikes in time bin}) / (\text{length of time bin})$$

Related to the probability a cell will spike (fire) in a given time interval.

Typically consider 50-70ms time bins.



Estimating Firing Rate



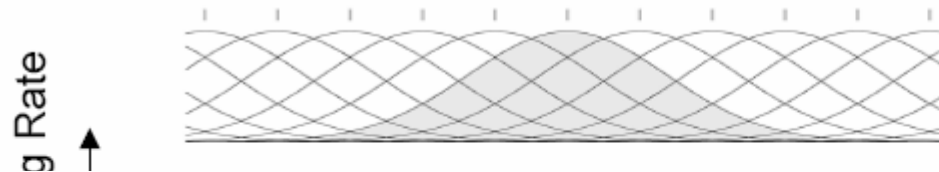
Source: Zemel & McNaughton, NIPS2000 tutorial

$$\text{rate} = 1/E(T)$$

where $E(T)$ is the "expected" time since the last spike.
The expectation can be computed using a causal window weighting function



Population Coding



- Broader tuning function always decreases the information carried by a single neuron
- Broader tuning function also implies that more neurons are activated at the same time so that the total information carried by the whole population often increases
- The accuracy of extracting information from a population is limited by the shapes of tuning functions and the probability distributions of noisy neuronal spikes



Population Rate Codes

- Cells modulate firing rate according to some tuning function.
- Groups of cells have different tuning functions that cover some “space”.



What are we talking about?

- Encoding or decoding?
- Causal?
- Generative?

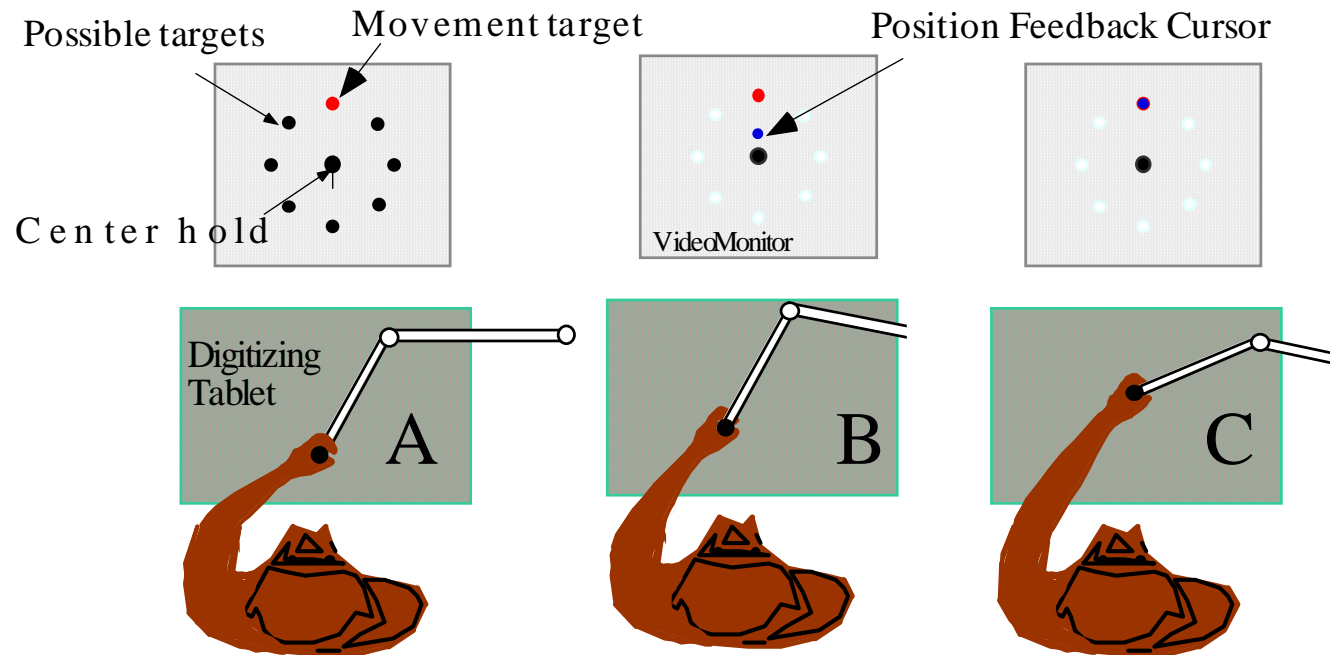


Elucidating the Encoding Model

- Simple tasks – find neural correlates.
- Stick an electrode into the brain and listen.



Center-Out Task

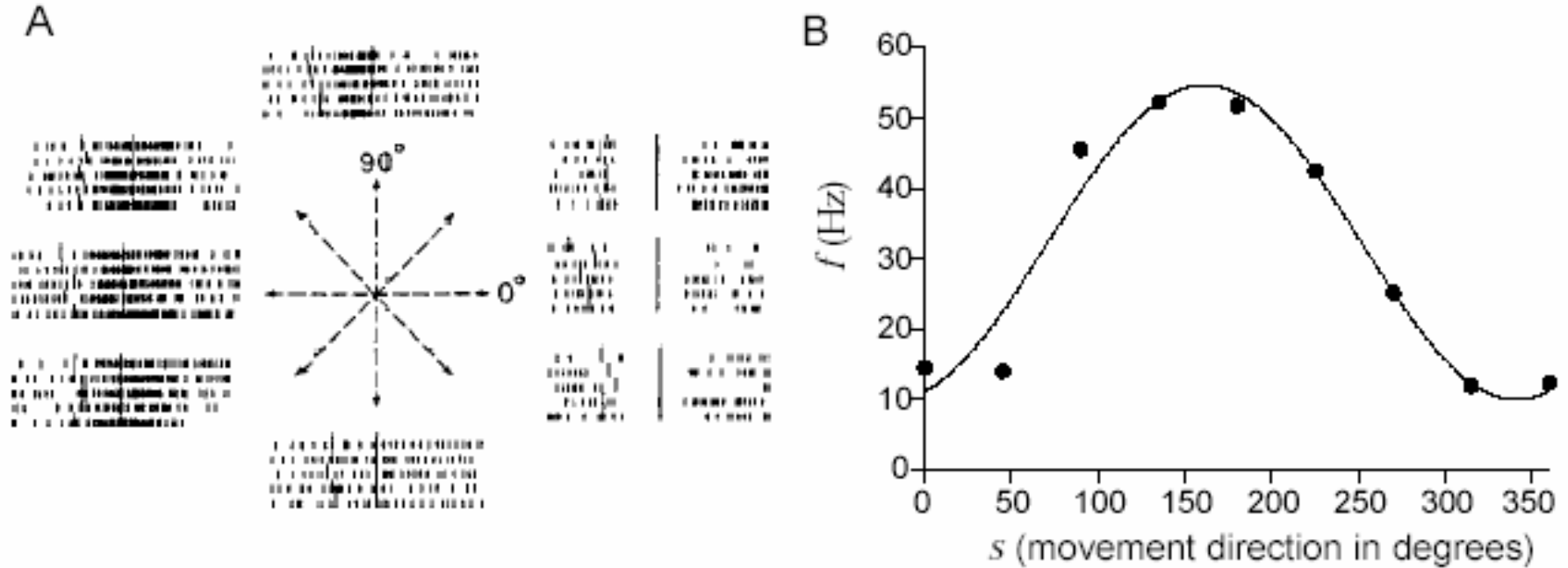


Move to center of screen, hold, target circle appears, move to target and hold.

Georgopoulos, Schwartz, & Kettner, '86. Moran & Schwartz, '99



Single Unit Center Out Encoding



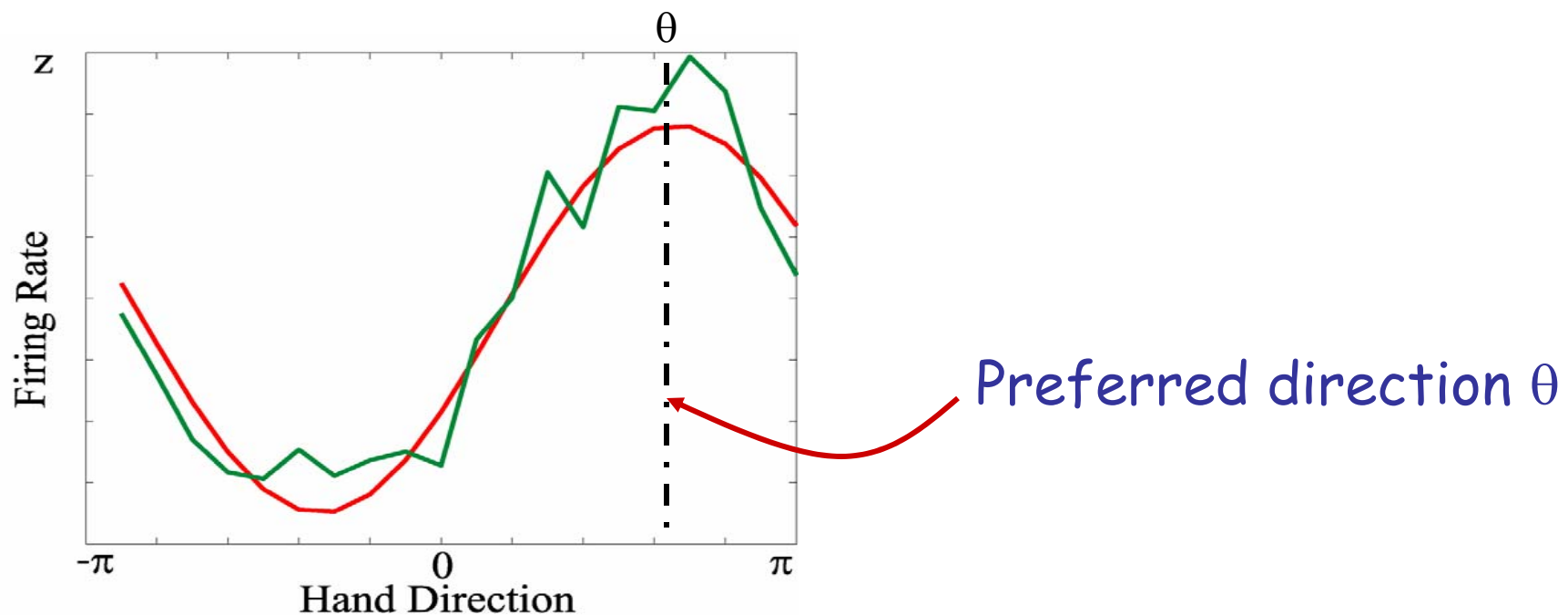


Motor Cortex Encoding

Georgopoulos et al ('82): (*cosine tuning* of single cells)

$$z_k = h_0 + h \cos(\theta_k - \theta)$$

z_k = firing rate, θ_k = hand direction



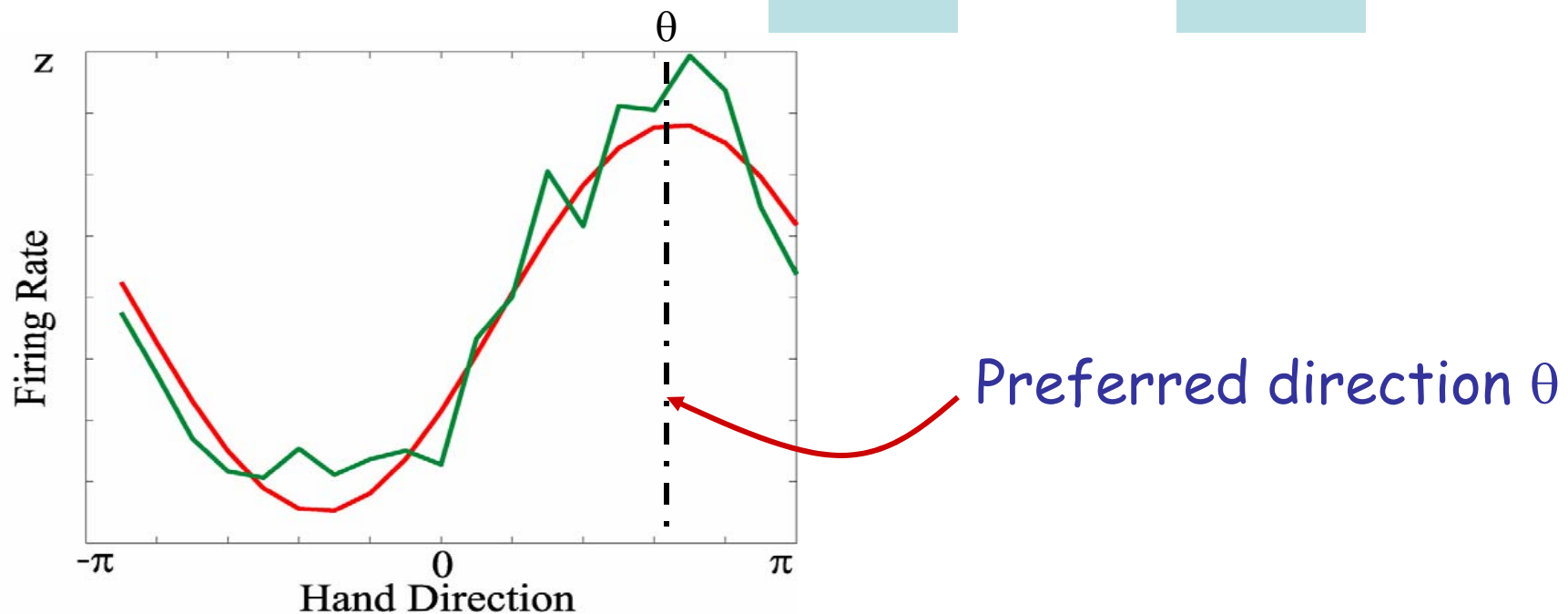


Encoding

Georgopoulos et al ('82): (*cosine tuning* of single cells)

$$z_k = h_0 + h \cos(\theta_k - \theta)$$

Recall: $\cos(\theta_k - \theta) = \cos \theta_k \cos \theta + \sin \theta_k \sin \theta$

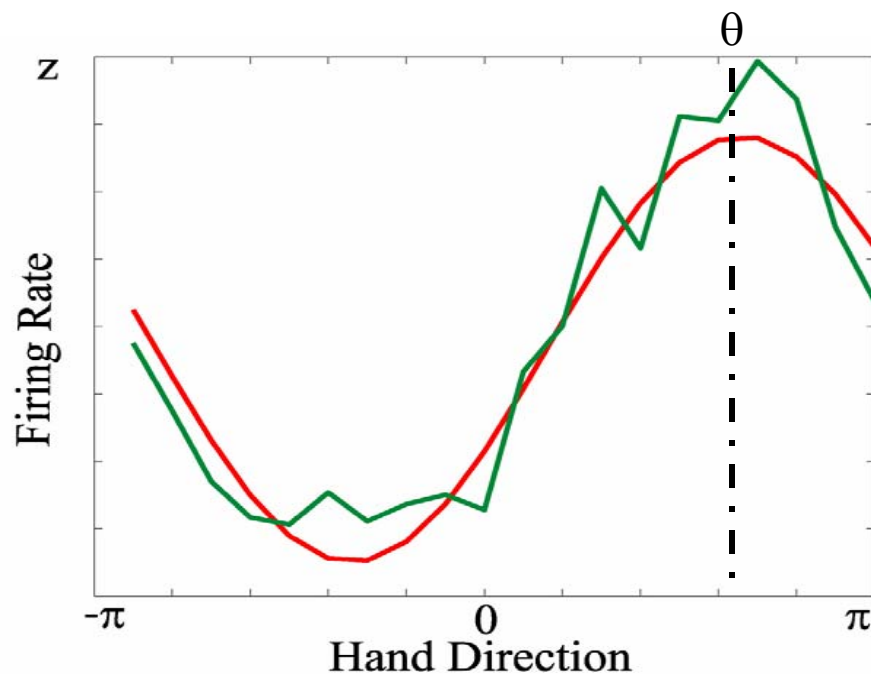




Encoding

Georgopoulos et al ('82): (*cosine tuning* of single cells)

$$\begin{aligned}z_k &= h_0 + h \cos(\theta_k - \theta) \\ &= h_0 + h_x \cos(\theta_k) + h_y \sin(\theta_k)\end{aligned}$$





On to decoding....

- Population vectors
 - Discuss Moran & Schwartz
- Matlab intro



More Talking Points

- Multiple parameters can be contained in the activity of single cells?
- How about the experiment design?
- What's histological examination?
- How about the math?
- What lag? Why?
- What about the prior models for motor cortex planning / movement coordination?
- Extensibility?



Main Points

- Reaching is achieved through continuous dynamic time-varying correlations between cortical activity and arm movement.
- A single equation relating motor cortical discharge rate to average directional selectivity and time-varying speed of movement was developed for reaching.
- The authors formulate a single equation which relates motor cortical discharge rate to directional selectivity and speed of movement in center-out tasks.

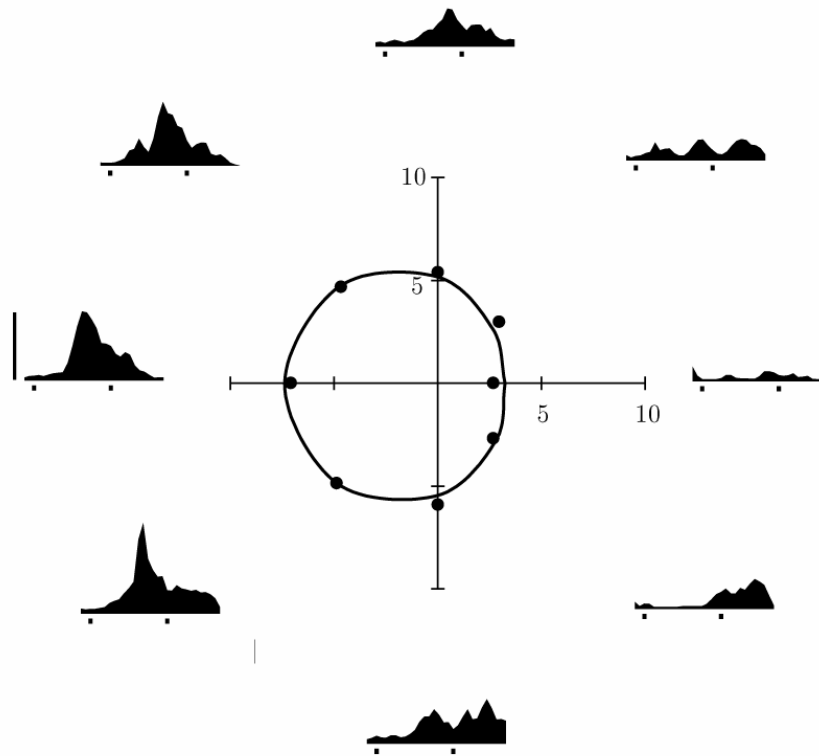


Open Questions

- How do constants b_0 , b_x , b_y vary across subjects ? Can these parameters be predicted accurately for different subjects of the same species or must they be tweaked for each user. On a related note, how do these parameters vary across species ?
- - Why does acceleration not affect discharge rate of M1 cells ? Quick physical movement necessitating acceleration (and hence power consumption) can be important at times. Are motor cells not responsible for these ? Do some other cells completely take over in such a case ?
- - Eq1 has a deterministic formulation. Does this mean that we've completely captured the behaviour of the cell ? Is there any randomness or uncertainty in the firing behavior of a cell ?
- - Following Fig 15D, Pmd cells do not follow eq1 completely. Does a variable lag account for the discrepancy ?
- All neural activity was obtained by implanting electrodes into the brain, so one direction for future research would be to obtain the same readings in a non-invasive form. Also, the trials were done on monkeys and would need to be repeated on human subjects to be of any practical use.
- Would the results be the same for leg control or fine control like grasping and finger flexion?
- So, maybe in future, the tests can be conducted in a more surprising way for the monkeys so that they won't do the movement automatically, and the results in that case should be examined.



Decode from Single-Cell Activity



Moran & Schwartz, '99

$$\bar{f}_j^{1/2} = b_0 + b_x \sin(\bar{\theta}_j) + b_y \cos(\bar{\theta}_j)$$

Single cells from multiple animals.

Average rate over RT and MT to each target (300-600 ms).

Fit with cosine encoding model.

Infer firing conditioned on speed by assuming a bell-shaped function and factoring out direction effects.



Cosine Tuning Curve

For reaching direction given by vector \mathbf{r} or angle θ , the firing rate above baseline for neuron i is

$$a_i = \mathbf{p}_i \cdot \mathbf{r} = |\mathbf{p}_i| |\mathbf{r}| \cos(\theta - \theta_i)$$

where vector \mathbf{p}_i and angle θ_i describe its preferred direction.

Population Vector

Population vector is defined by

$$\mathbf{v} = \sum_i a_i \mathbf{p}_i$$

It is a linear combination of the preferred directions weighted by the activity of each neuron.



Principle of Population Vector

Population vector always recovers the true reaching direction :

$$\mathbf{v} = \mathbf{r}$$

if and only if

$$\sum_i \mathbf{p}_i \mathbf{p}_i^T = \mathbf{I}$$

where \mathbf{I} is the identity matrix, and the preferred direction \mathbf{p}_i is taken as a column vector with T indicating transpose.

This is because $\mathbf{v} = \sum_i \mathbf{p}_i a_i = \sum_i \mathbf{p}_i (\mathbf{p}_i^T \mathbf{r}) = \left(\sum_i \mathbf{p}_i \mathbf{p}_i^T \right) \mathbf{r}$

where the 1st step is a definition and the 2nd step is by cosine tuning.

A special case that satisfies the identity matrix condition is a population of uniformly distributed preferred directions, which is roughly true in the motor cortex.



Basis Functions

Population vector is equivalent to finding the peak in the linear combination of cosine functions weighted by activity a_i of each neuron :

$$\sum_i a_i \cos(\theta - \theta_i)$$

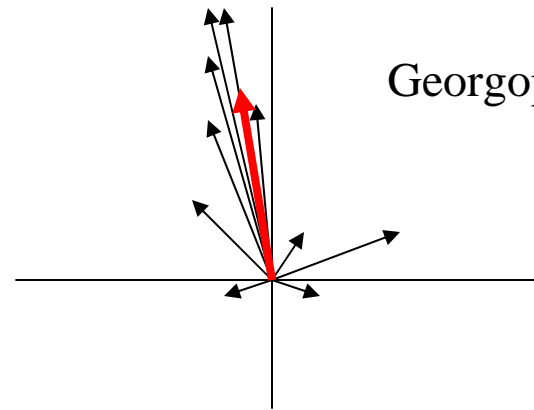
More generally, using other basis function instead of the cosine, we write

$$\sum_i a_i \Phi_i(\theta)$$

There are many choices of the basis functions. The result can be much more accurate than the population vector when suitable basis functions are used.



Population Vectors

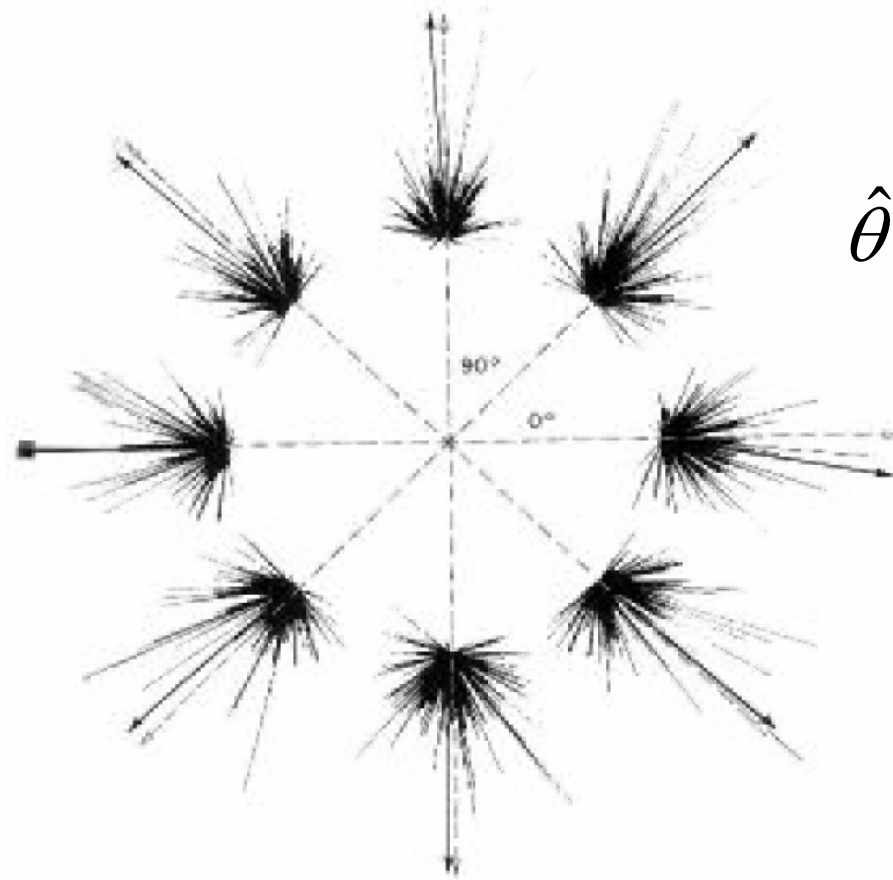


Georgopoulos, Schwartz & Vetter '86

- Take each cell's "preferred" direction and weight it by its current activity.
- Summing all the weighted directions gives some measure of the current direction.
- Populations computed from *multiple animals*.



Population Vector



$$\hat{\theta} = \sum_i r_i \bar{\theta}_i$$

Source: Kandel, Schwartz, and Jessell



Our Data

- Continuous motion (can't show movie)
- Firing rates of 42 cells
- Matlab demo