Homework 2

Due: 2:30 PM Feb. 28, 2008

To submit your code electronically, make sure that in your `home` directory you have a `course` directory and that in that there is a `cs004` directory. You should save all your programs into `.m` files. For this assignment, you will place all your code into the directory

```
/u/<login>/course/cs004/hw2
```

(*Change* `<login>` *to be your login name.*) After you have your code saved in the proper directory you can run the handin script. Open a terminal, then type

```
cs004_handin hw2
```

The following commands will be useful for completing the homework. You can learn more about them by entering `help x` into the MATLAB command window, where `x` is the command.

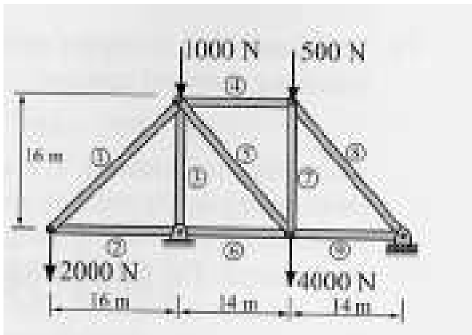
```
exp, cosd, sind, repmat, sum
```

Problem 2.1

Radioactive decay of radioactive materials can be modeled by the equation $A = A_0e^{kt}$, where A is the amount at time t, A_0 is the amount at $t = 0$, and k is the decay constant ($k \leq 0$). Technetium-99 is a radioisotope that is used in imaging of the brain. Its half-life is 6 hours. Calculate the relative amount of Technetium-99(A/A_0) in a patient's body for 24 hours after receiving a dose. After determining the value of k, define a vector $t = 0, 2, 4, \ldots, 24$ and calculate the corresponding values of A/A_0. Put your answer in a script file called **radioactive.m**.

Problem 2.2

A truss is a structure made of members jointed at their ends. For the truss shown in the figure, the forces in the nine members are determined by solving the following system of nine equations.



$$
\begin{aligned}
cos(45°)F_1 + F_2 &= 0 \\
F_4 + cos(48.81°)F_5 - cos(45°)F_1 &= 0 \\
-sin(48.81°)F_5 - F_3 - sin(45°)F_1 &= 1000 \\
cos(48.81°)F_8 - F_4 &= 0 \\
-sin(48.81°)F_8 - F_7 &= 500 \\
F_9 - cos(48.81°)F_5 - F_6 &= 0 \\
F_7 + sin(48.81°)F_5 &= 4000 \\
sin(48.81°)F_8 &= -1107.14 \\
-cos(48.81°)F_8 - F_9 &= 0
\end{aligned}
$$

Write the equations in a matrix form and use MATLAB to determine the forces in the members. A positive force means tensile force and a negative force means compressive force. Display the results as a column vector (make sure to specify which variable corresponds to which row). Put your answer in a script file called **truss.m**.

Problem 2.3

The position as a function of time $(x(t), y(t))$ of a projectile fired with a speed of v_0 at an angle θ is given by

$$
\begin{aligned}
x(t) &= v_0 \cos\theta \cdot t \\
y(t) &= v_0 \sin\theta \cdot t + \frac{1}{2}gt^2
\end{aligned}
$$

where $g = -9.81$ m/s^2 is the acceleration due to gravity. The distance r to the projectile at time t can be calculated by $r(t) = \sqrt{x(t)^2 + y(t)^2}$. Consider the case where $v_0 = 100$ m/s and $\theta = 79$ degrees. Determine the distance r to the projectile for $t = 0, 2, 4, \ldots, 20$ s. Put your answer in a script file called **projectile.m**.

Problem 2.4

Now we're going to teach you how to make money. . . Well, not really—but we'll do an exercise that involves quite a bit of money. Let's say Itay had \$100,000—a fair sum. He decided he wanted to invest it in AT&T, a solid, well-managed firm. Before he puts his money down though, he wants to do some research; he believes the price trends for 2008 will be similar to those for 2005. Let's assume that there are only 252 days in 2005, because that's the data we have. Itay is debating between two strategies, each of which assumes that he's allowed to buy fractional amounts of stock.

Strategy 1: Invest the \$100,000 as a lump sum on the first day(1/1/05) and sell it on all the last day (12/31/05). By "invest", we mean "buy 100,000 dollars-worth of stock at the closing-price on the first day."

Strategy 2: Invest the same dollar amount each day (so \$100,000 divided by 252) by buying as much stock as you can with that sum, and then selling it all at the end of the year (12/31/05).

You're going to help Itay find out which is better. We've provided a file for you, `att.csv`, which holds the closing prices for AT&T for this period (1/1/05-12/31/05). If you are working in the Sunlab, you can find it at `/course/cs004/pub/hw2/att.csv`. If you are working at home, you can find it on the course website. As mentioned above, there are only 252 days in our year - the rest are a long holiday. You have several tasks:

Task 1: Find out how much money you would have ended up with using strategy 1. This is essentially a one line task in MATLAB, though you may wish to break it up. If you do break it up, it should be no more than 4 lines. (Hint: remember `end`.)

Task 2: Find out how much money you would have ended up with using strategy 2. This is essentially a one line task in MATLAB, though you may wish to break it up (should be no more than 5 lines). (Hint: you may want to use `repmat` to get a matrix of your daily spending money).

Task 3: Which strategy worked better? Are you surprised? (Answer those questions.) Now find out how many days have prices that are less than or equal to the price on 1/1/05. Does this explain why?

Put your answer in a script file called **att.m**.

Problem 2.5

Find all the primes between 1 and 1000 in 2 lines or less (Hint: use `isprime`.) Put your answer in a script file called **primes.m**.

Problem 2.6

A ball that is dropped on the floor bounces back up many times reaching a lower height after each bounce. When the ball impacts the floor its rebound velocity is 0.85 times the impact velocity. The velocity v at which a ball hits the floor after being dropped from a height h is given by $v = \sqrt{2gh}$, where $g = 9.81$ m/s^2 is the acceleration of the Earth. The maximum height h_{max} that a ball reaches is given by $h_{max} = \frac{v^2}{2g}$, where v is the upward velocity after impact. Consider a ball that is dropped from a height of 2m. Determine the height the ball reaches after the first 8 bounces. (Calculate the velocity of the ball when it hit the floor for the first time. Derive a formula for h_{max} as a function of the bounce number. Then create a vector $n = 1, 2, \ldots, 8$ and use the formula (use element-by element operations) to calculate a vector with the values of h_{max} for each n.) Put your answer in a script file called **ball.m**.

Problem 2.7

Two projectiles, A and B, are shot at the same instant from the same spot. Projectile A is shot with a velocity of 680 m/s at an angle of 65° and projectile B is shot at a velocity of 780 m/s at an angle of 42°. Determine which projectile will hit the ground first. Then, take the flying time t_f of that projectile and divide it into ten increments by creating a vector *time* with 10 equally spaced elements (the first element is $\frac{t_f}{10}$, the last is t_f) Using the time vector calculate the x and y positions of each bullet at each time. Then calculate the distance between the two projectiles at the ten timesteps in vector t. Put your answer in a script file called **twoprojectiles.m**. The following equations are given to help. Assume $g = -9.8\frac{m^2}{s}$.

$$z = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \tag{1}$$

Where z is equal to the distance between ordered pairs x_1, y_1 and x_2, y_2.

$$t_f = \frac{-2v\sin\theta}{g} \tag{2}$$

Flying time of a projectile.

$$x(t) = vt\cos\theta \tag{3}$$

x position as a function of time.

$$y(t) = vt\sin\theta + \frac{1}{2}gt^2 \tag{4}$$

y position as a function of time. In both (3) and (4), t is time, and v and θ refer to the initial velocity and angle of the projectiles.