Homework 5

Due: 2:30 PM Mar. 20, 2008

To submit your code electronically, make sure that in your `home` directory you have a `course` directory and that in that there is a `cs004` directory. You should save all your programs into `.m` files. For this assignment, you will place all your code into the directory

`/u/<login>/course/cs004/hw5`

(*Change* `<login>` *to be your login name.*) After you have your code saved in the proper directory you can run the handin script. Open a terminal, then type

`cs004_handin hw5`

The following commands will be useful for completing the homework. You can learn more about them by entering `help x` into the MATLAB command window, where `x` is the command.

`meshgrid, surf, max, ode45, max, cumtrapz`

Problem 5.1

The Verhulst model, given in the following equation, describes the growth of a population that is limited by various factors such as overcrowding, lack of resources, etc.:

$$N(t) = \frac{N_\infty}{1 + (\frac{N_\infty}{N_0} - 1)e^{-rt}}$$

where $N(t)$ is the number of individuals in the population, N_0 is the initial population size, N_∞ is the maximum population size due to the various limiting factors, and r is a rate constant. Make a surface plot of $N(t)$ versus t and N_∞ assuming $r = 0.1\ \mathrm{s}^{-1}$, and $N_0 = 10$. Let t vary between 0 and 100 and N_∞ between 100 and 1000. Save your solution in an m-file called `verhulst.m`, and be sure to label the axes and save your figure as `verhulst.fig`.

Problem 5.2

Planck's radiation law gives the spectral radiancy R as a function of the wave length λ and temperature T (in degrees K):

$$R = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

where $c = 3.0 \times 10^8 m/s$ is the speed of light, $h = 6.63 \times 10^-34$ J-s is the Planck constant, and $k = 1.38 \times 10^-23$ J/K is the Boltzmann constant.

Plot R as a function of λ for $0.2 \times 10^-6 \leq \lambda \leq 6.0 \times 10^-6$m at $T =$ 1500 K, and determine the wavelength that gives the maximum R at this temperature. Mark the location of this λ value on your plot. Make sure to label your axes and give your plot a title and a legend as well, and save your solution in an m-file called `radiancy.m`.

Problem 5.3

The sudden outbreak of an insect population can be modeled by the equation:

$$\frac{dN}{dt} = RN\left(1 - \frac{N}{C}\right) - \frac{rN^2}{N_c^2 + N^2}$$

The first term relates to the well-known logistic population growth model, where N is the number of insects, R is an intrinsic growth rate, and C is the carrying capacity of the local environment. Its effect becomes significant when the population reaches a critical size N_c. r is the maximum value that the second term can reach at large values of N.

Create an m-file called `insectgrowth.m` to solve the differential equation. Use $0 \leq t \leq 50$ days and two growth rates, once for $R = 0.55$days$^-1$ and once for $R = 0.58$days$^-1$. Solve equations with $N(0) = 10,000.$, $C = 10^5$, $N_c = 10^4$, and $r = 10^4$. Make one plot comparing the two solutions, and in the comments to `insectgrowth.m`, discuss why this model is called an "outbreak" model. Save your figure as `insectgrowth.fig`.

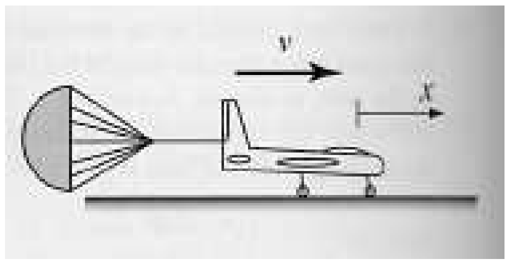
Problem 5.4

An airplane uses a parachute and other means of braking as it slows down on the runway after landing. Its acceleration is give by $a = -0.0035v^2 - 3$m/s^2.

Since $a = \frac{dv}{dt}$, the rate of change of the velocity is given by:

$$\frac{dv}{dt} = -0.0035v^2 - 3$$

Consider an airplane with a velocity of 300 km/h that opens its parachute and starts decelerating at $t = 0$ s.



a) By solving the differential equation, determine and plot the velocity as a function of time from $0 \leq t \leq 15$ s until the airplane stops (i.e. V=0). Be sure to label your graph with the appropriate information (title, legend, dependant variables, and independant variables).

b) Use numerical integration to determine the distance x the airplane travels as a function of time (hint: use the cumtrapz function in matlab). Make a plot of x vs. time, that will appear on the same figure as the plot you just constructed.

Write an anonymous function for acceleration to solve with a matlab ODE solver, and put your solution in an m-file called `airplane.m`. Make sure you save the resulting figure as `airplane.fig`.