Lab 5

Out: March 9, 2008

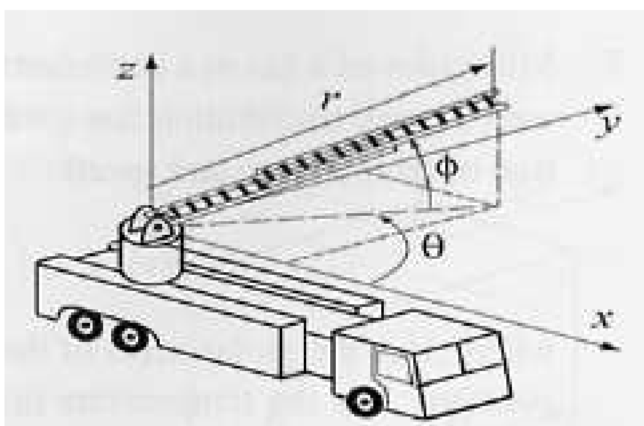
Make sure to check on the screen in the front for your name. You will need an email from a Head TA to get credit for this lab if you are not on the list.

Important functions: ode45, surf, meshgrid, sph2cart

Please use an anonymous function for problem 5.3

Problem 5.1

The ladder of a fire truck can be elevated (increase of angle ϕ), rotated about the z axis (increase of angle θ), and extended (increase of r). Initially the ladder rests on the truck, $\phi = 0$, $\theta = 0$, and $r = 8$ m. Then the ladder is moved to a new position by raising the ladder at a rate of 5 deg/s, rotating at a rate of 8 deg/s, and extending the ladder at a rate of 0.6 m/s. Determine and plot the position of the tip of the ladder for 10 seconds. The position is related to phi, theta, and r. (This can be done in Cartesian or spherical coordinates. See helpful functions)



Problem 5.2

The van der Waals equation gives a relationship between the pressure p (in atm.), volume V, (in L), and temperature T (in K) for a real gas:

$$P = \frac{nRT}{V - nb} - \frac{n^2 a}{V^2}$$

where n is the number of moles, $R = 0.08206$ (L atm)/(mole K) is the gas constant, and a (in L^2 atm/mole^2), and b (in L/mole) are material constants.

Consider 1.5 moles of nitrogen ($a = 1.39$ L^2atm/mole^2, $b = 0.03913$ L/mole). Make a plot that shows the variation of pressure (dependent variable, z axis) with volume (independent variable, x axis) and temperature (independent variable, y axis). (This plot will have two independent variables. This forces you to use a specific type of plot.) The domains for the volume and temperature are: $0.3 \leq V \leq 1.2$ L, and $273 \leq T \leq 473$ K.

Problem 5.3

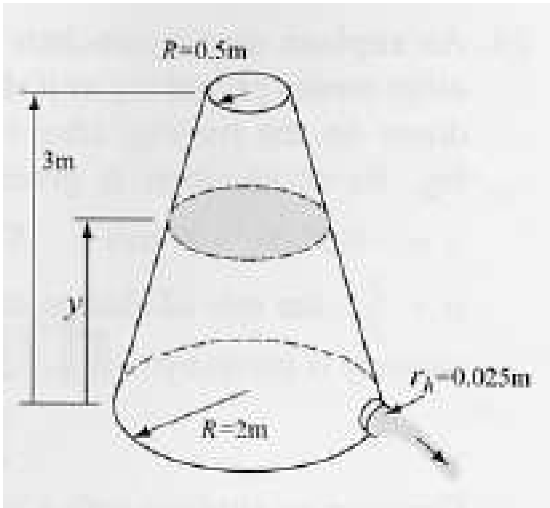
A water tank shaped as an inverted frustum cone has a circular hole at the bottom on the side, as shown. According to Torricelli's law, the speed v of the water that is discharging from the hole is given by

$$v = \sqrt{2gy}$$

where y is the height of the water and $g = 9.81m/s^2$. The rate at which the height, y, of the water in the tank changes as the water flows out through the hole is given by:

$$\frac{dy}{dt} = \frac{-v * r_h^2}{(2 - 0.5y)^2}$$

where r_h is the radius of the hole.



Complete the following steps in a single m file.

1) Make an anonymous function "dydt" for the given equation (your anonymous function should take t and then y as its arguments).

2) Then solve this ordinary differential equation (ODE) over the time span $t = 0$ s to 3000 s, using an initial water height of $y = 2$ m. (Hint: look at last Thursday's lecture.)

3) Find the index of where the height first falls below 0.1 m, and, using only the parts of your time and height vectors up to this index, plot the height of the water as a function of time.

4) Take a look at your solved vector of heights; you'll notice that the values after a certain point become imaginary. Why might this be happening? (Hint: what values of the variable(s) would make the given equation imaginary, and is the the function you used to solve the ODE using an exact solution or an approximation?)