CSCI 0220 **Discrete Structures and Probability** B. Raphael

Homework 2

Due: 15 Feb 2008

Reading: from S. Epp, *Discrete Mathematics with Applications*, Sections 2.2-2.4; 3.1-3.3

Problem 2.1

Rewrite each of these statements so that negations appear only applied to predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

(a) $\sim[\forall x \forall y P(x, y)]$

(b) $\sim[\forall y \exists x P(x, y)]$

(c) $\sim[\forall y \forall x (P(x, y) \vee Q(x, y))]$

(d) $\sim[(\exists x \exists y \sim P(x, y)) \wedge (\forall x \forall y Q(x, y))]$

(e) $\sim[\forall x (\exists y \forall z P(x, y, z) \wedge \exists z \forall y P(x, y, z))]$

Problem 2.2

Let $S(x, y)$ be the statement "x can apprehend y," where the domain is all people. Use predicates and quantifiers to express the following statements:

a. "All V. I. L. E. agents can be apprehended by somebody."

b. "There is exactly one person whom everybody can apprehend."

c. Express the following statement by two different but equivalent logical expressions using what you know about propositional logic:
"No one can apprehend both Carmen Sandiego and the Contessa."

d. Now demonstrate that your two statements in part c are logically equivalent.

Problem 2.3

Show that the sum of even integers is even.

Problem 2.4

Given any integers a, b, c, if $a - b$ is odd and $b - c$ is even, what can you say about the parity (odd/even) of $a - c$? Support your answer with a proof.

Problem 2.5

Our fancy base 10 number system has an interesting property: a number is divisible by 3 if and only if the sum of its digits is divisible by 3. Show why this is true.

Problem 2.6

Prove or give a counterexample:

a. For all integers a, b, and c, if $a|bc$ then $a|b$ or $a|c$.

b. For all primes a and all integers b and c, if $a|bc$ then $a|b$ or $a|c$.

c. For all integers a and b, $a|b$ if and only if $a^2|b^2$..

The following problems are *non-collaborative*—discuss them with no one but the professor and the TAs.

Non-collaborative Problem 2.7

Give the contrapositive, converse, and inverse of the following statements.

a. $\forall d \in \mathbb{Z}, \frac{6}{d} \in \mathbb{Z} \rightarrow d = 3$.

b. $\forall n \in \mathbb{Z}$, if n is prime, then $n = 2$ or n is odd.

c. If the square of an integer is odd, then the integer is odd.

Non-collaborative Problem 2.8

Reorder the given premises and use contraposition to show that the conclusion follows from the premises.[1]

1. When I work a logic example without grumbling, you may be sure it is one I understand.
2. The arguments in these examples are not arranged in regular order like the ones I am used to.
3. No easy examples make my head ache.
4. I can't understand examples if the arguments are not arranged in regular order like the ones I am used to.
5. I never grumble at an example unless it gives me a headache.

$\therefore$ These examples are not easy.

[1]Adapted from Lewis Carroll, *Symbolic Logic* (New York: Dover, 1958), p. 123.