CSCI 0220 **Discrete Structures and Probability** B. Raphael

Homework 3

Due: Friday, 22 Feb 2008

Reading: S. Epp, Sections 3.4 – 3.7; 4.1 – 4.3.

Problem 3.1

Prove the following statement by contradiction:

For all real numbers x and y, if x is irrational and y is rational then $x - y$ is irrational.

Problem 3.2

a. Prove that for all integers a, if a^3 is even then a is even.

b. Prove that $\sqrt[3]{2}$ is irrational.

Problem 3.3

On the outside rim of a circular disk the integers from 1 through 30 are painted in random order. Show that no matter what this order is, there must be three successive integers whose sum is at least 45.

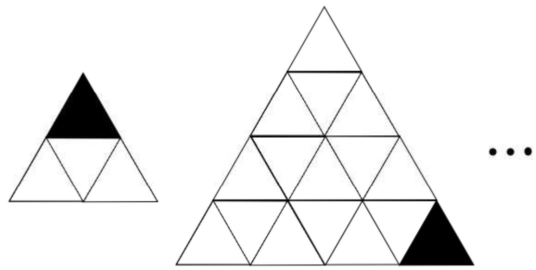
Problem 3.4

Prove the following using induction:

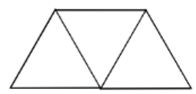
$6 \mid n(n^2 + 5), \forall n \geq 1$

Problem 3.5

Consider the set of equilateral triangles with sides of length 2^n, where $n > 0$, with one 1 x 1 x 1 triangle $\triangle$ missing from *any* one of the three corners.



Show that each figure in the set can be covered with tiles composed of three 1 x 1 x 1 triangles △ in the form:



Problem 3.6

Using induction, prove that

$$\sum_{k=1}^{n+1} k \cdot 2^k = n \cdot 2^{n+2} + 2 \quad \forall n \geq 0$$

The following problem is *non-collaborative*—discuss it with no one but the professor and the TAs.

Non-collaborative Problem 3.7

Prove by induction that

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2} \right)^2$$

BONUS Problem:

Problem 3.8

The following is a Bonus Problem.

Suppose that n chords are drawn on a circle in such a way that each chord intersects every other, but no three intersect at one point. Prove that the chords divide the circle into $\frac{n^2+n+2}{2}$ regions. The following example shows the case $n = 3$, where the circle is divided into 7 regions.