Homework 6

Due: Friday, 14 Mar 2008

All homeworks are due at 1:00pm in the CS22 bin on the CIT second floor, opposite the elevators.

Write your *full name* and the problem number on each piece of paper you hand in and then staple.

Reading: S. Epp, sections 10.5; 11.1–11.2

Problem 6.1

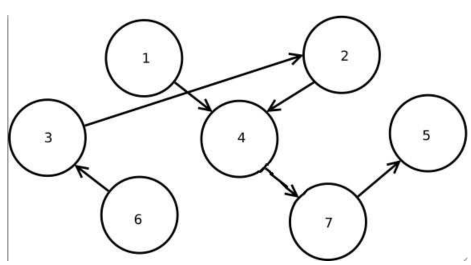
a. Let (S, R) be a poset. Show that (S, R^{-1}) is also a poset, where R^{-1} is the inverse relation of R. The poset (S, R^{-1}) is called the *dual* of (S, R).

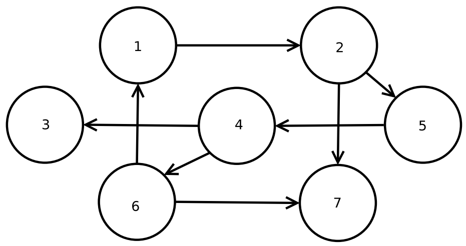
b. Suppose that $(S, \preceq_1)$ and $(T, \preceq_2)$ are posets. If $\preceq$ is defined by $(s, t) \preceq (u, v)$ if and only if $s \preceq_1 u$ and $t \preceq_2 v$ for $s, u \in S$, $t, v \in T$, show that $(S \times T, \preceq)$ is a poset

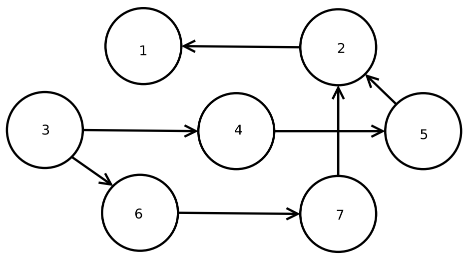
Problem 6.2

Solve for each of the following:

- Can the graph be sorted topologically?
- If so, does there exist a unique sort?
- If there is more than one sort, enumerate all possible sorts.

a. 

b. 

c. 

Problem 6.3

Let $G = (V, E)$ be a simple graph. Let R be the relation on V consisting of pairs of vertices (u, v) such that there is a path from u to v or such that $u = v$. Show that R is an equivalence relation.

Problem 6.4

The Brown CS department has decided to secede from Facebook, creating their own internal social network called CodeBook. For space considerations, the programmers need to know how many possible friendships will exist in this new network. Determine the $m$aximum total number of friendships that can exist in CodeBook if there are n people in the network, and prove your solution.

Problem 6.5

Prove that if G is a simple graph with at least two vertices, then G has two or more vertices with the same degree.

Problem 6.6

Let $S = \{1, 2, 3, 4\}$.

a) Draw the Hasse diagram for the poset $A = \mathcal{P}(S)$ with the subset partial order.

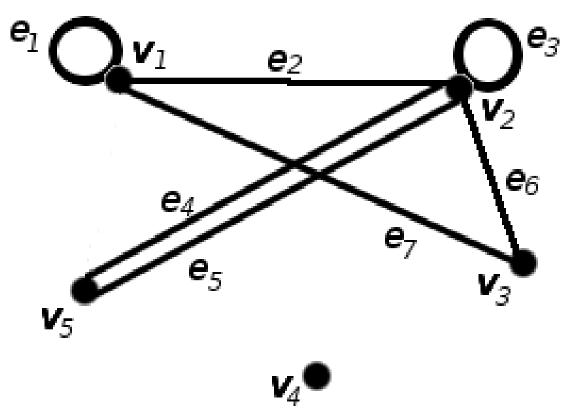
b) Find the smallest partition of A into chains. Prove that your answer is correct. (Note: you must show (i) your answer is a partition (ii) each element in your partition is a chain and (iii) there is no partition of A into fewer chains.)

c) Find the smallest partition of A into antichains. Prove that your answer is correct.

The following problem is *non-collaborative*—discuss it with no one but the professor and the TAs.

Non-collaborative Problem 6.7

a) In the following graph, vertices are denoted by v and edges by e. Do the following:



i. find all edges that are incident on v_1

ii. find all vertices that are adjacent to v_3

iii. find all edges that are adjacent to e_1

iv. find all loops

v. find all parallel edges

vi. find all isolated vertices

vii. find the degree of v_3

viii. find the total degree of the graph

For each of b-d), either draw a graph with the specified properties or explain why no such graph exists:

b) Graph with four vertices of degrees 1, 1, 1, and 4.

c) Graph with four vertices of degrees 1, 2, 3, and 4.

d) Simple graph with five vertices of degrees 1, 1, 1, 2, and 3.

Bonus Problem:

Problem 6.8

Suppose a deck of n^2+1 cards labeled 1 through n^2+1 is randomly shuffled. A dealer deals the deck into two rows on a table, each time placing the next card in the deck face up at the end of one of the rows. Show that the dealer can always deal the cards so that one of the rows contains $n+1$ cards in increasing OR decreasing order. (Mathematically, this means that the deck of cards contains a increasing subsequence OR a decreasing subsequence.) *Hint:* Use a theorem relating sizes of chains and antichains given in class.