Homework 7
More Predicate Logic (and some DT stuff)

Due: 5:00pm on 4/07/08

Problem 7.1

Read chapter 14 from Russel and Norvig.

Problem 7.2

You can represent a queue as an abstract data structure in predicate logic. The following axioms define basic operations like enqueue, dequeue, and first on a queue:

a. $Queue(\epsilon)$

b. $\forall x : Queue(x) \Rightarrow x = \epsilon \vee \exists y, z : Queue(y) \wedge x = Enqueue(y, z)$

c. $\forall x, y : Queue(x) \Rightarrow Queue(Enqueue(x, y))$

d. $\forall y : Front(Enqueue(\epsilon, y)) = y$

e. $\forall x, y : Queue(x) \Rightarrow x = \epsilon \vee Front(x) = Front(Enqueue(x, y))$

f. $\forall y : Dequeue(Enqueue(\epsilon, y)) = \epsilon$

g. $\forall x, y : Queue(x) \Rightarrow x = \epsilon \vee Dequeue(Enqueue(x, y)) = Enqueue(Dequeue(x), y)$

Using the above as the basis for your queue:

a. Give the axioms for a function, Append(x,y) that takes the contents of one queue and appends it onto queue in predicate logic.

   Solution:

   $\forall x, y : Queue(x) \wedge Queue(y) \Rightarrow Append(x, y) = Append(Enqueue(x, Front(y)), Dequeue(y))$

   $$Append(x, \epsilon) = x$$

b. Give the axioms for Reverse(x), which reverses the order of the elements in a queue, in predicate logic using Append from part a.

   Solution:

   $$Reverse(\epsilon) = \epsilon$$

   $$\forall x, y : Queue(x) \Rightarrow Reverse(Enqueue(x, y)) = Append(Enqueue(\epsilon, y), Reverse(x))$$

Problem 7.3

a. Transform the following into Skolem normal form:

$$\alpha = \forall x \forall y \forall z [f(f(x,y),z) = f(x, f(y,z)) \wedge \exists e \forall x (f(x,e) = x \wedge \exists x' (f(x,x') = e))]$$

Solution:

$$\alpha = \forall x \forall y \forall z [f(f(x,y),z) = f(x, f(y,z)) \wedge (f(x,g(x)) = x \wedge (f(x,h(x)) = g(x)))]$$

b. Unify the following atomic sentences or give an argument why this is not possible. P is a predicate, a and b are constants, f, g and h are functions and x, y, z, x_0, x_1, x_2 are variables. Assume that standardizing apart has already been performed, i.e., unify the pair of sentences without performing any variable renaming.

(a) $\{p(f(f(f(a,x_0),x_1),x_2)),$
$p(f(x_2, f(x_1, f(x_0, a))))\}$

Solution:

$$x_2/f(f(a,x_0),x_1), x_1/f(a,x_0), x_0/a$$

(b) $\{p(x),$
$p(f(x))\}$

Solution: Occurs check.

(c) $\{p(x, h(x,y), y),$
$p(x, z, b)\}$

Solution:

$$z/h(x,y), y/b$$

(d) $\{p(x,x,y,y),$
$p(x, f(y), y, f(x))\}$

Solution: Occurs check.