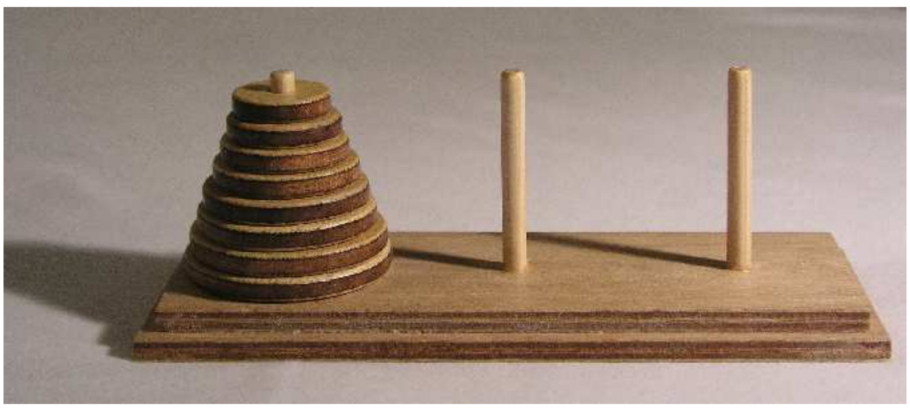
Homework 0
Warmup

Due: 6:00pm on 09/16/09

Problem 0.1

Turn in your collaboration policy to a TA on hours, or to Yuri's office (CIT 409).

Problem 0.2



The Tower of Hanoi puzzle was invented in 1883. You are given three pegs and three disks which are initially stacked in decreasing size on the left peg. The object of the puzzle is to recreate the stack on the right peg while observing two restrictions: you can only move one disk at a time, and a larger disk can never be placed on top of a smaller disk.

a) What are the possible states after 2 moves? after 3 moves?

b) What is the optimal (i.e. fewest number of moves) solution?

c) Argue that your solution is optimal?

d) Are there other optimal solutions (i.e. same number as moves as your first solution)?

e) *Extra Credit:* It is said that only a god can solve the thirty-four disk, three peg Towers of Hanoi puzzle. Given one second per move, how long would it take you complete the puzzle?

Solution:

We can represent the tower as a comma delimited list of numbers. The smaller the number the smaller the disk, so the initial state would be (1 2 3, ,)

a) Two moves: (1 2 3, ,), (2 3, 1,), (2 3, , 1), (3, 1, 2), (3, 2, 1)
Three moves: (2 3, 1,), (2 3, , 1), (1 3, 2,), (1 3, , 2), (3, 1 2,), (3, , 1 2), (3, 2, 1), (3, 1, 2), (123, ,)

b) (1 2 3, ,) -> (2 3, , 1) -> (3, 2, 1) -> (3, 1 2,) -> (, 1 2, 3) -> (1, 2, 3) -> (1, , 2 3) -> (, , 1 2 3)

c) The only ways for us to get 3 from the left peg to the right from disk arrangements (3, 1 2,) or (1 2, 3,). From part (a), we see that (3, 1 2,) can be reached in 3 steps, and no fewer. (1 2, 3,) cannot be reached as quickly. So, so we know, when we have arrangement (, 1 2, 3), we cannot have made fewer than 4 moves. Because the two steps of moving the 3 are symmetric, we need only consider the shorter path, through (3, 2 1,). We now need to get the 2 onto the right peg. We need to get into position (1, 2, 3) or (2, 1, 3); (1, 2, 3) is reachable in one move. That brings us to (1, , 2 3), which is one move away from the solution, (, , 1 2 3). This is somewhat "hand-wavy"; a complete proof could be done by fully enumerating all possible moves.

d) No. We know there is only one quickest way to get 3 onto the right peg from part (a), and, due to symmetry, that no longer way of getting 3 to the right peg will yield a better solution. From the (, 1 2, 3) position, we could (but won't) construct a tree showing all possible combinations of the next three moves, which would show only one sequence that leads us to the goal.

e) 548 years.

Problem 0.3

A new disease has been discovered, but the test for it, while accurate, is very expensive. A doctor is trying to determine if people have the disease by only looking at a few symptoms. He has data from his last few patients who have taken the expensive test. Here are the results:

	Cough	*TummyAche*	*SoreThroat*	*Chills*	*HasDisease*
PatientA	*no*	*no*	*yes*	*yes*	*no*
PatientB	*yes*	*yes*	*no*	*no*	*no*
PatientC	*yes*	*yes*	*yes*	*no*	*yes*
PatientD	*no*	*no*	*yes*	*no*	*no*
PatientE	*no*	*yes*	*yes*	*yes*	*yes*

Now Patient F walks into the doctor's office and has the following symptoms:

	Cough	*TummyAche*	*SoreThroat*	*Chills*
PatientF	*no*	*yes*	*yes*	*no*

Do you think Patient F has the disease? why? (Hint: Your answer should be more than one word long)

Solution:

Everyone who has the disease has both a sore throat and a tummy ache, and everyone with both a sore throat and a tummy ache has the disease. Since Patient F has both symptoms, Patient F probably has the disease.

Problem 0.4

Matt only has three dollars but is determined to buy lunch on Thayer street. Yuri has four dollars and loves to gamble. Matt challenges Yuri to a simple game. They agree to flip a fair coin. If it's heads, Matt gives Yuri a dollar. If it's tails, Yuri gives Matt a dollar. They decide to stop either when one person has all seven dollars. What is the probability that Matt takes home seven dollars?

Solution:

Suppose we have states s_i, where i is the amount of money Matt posesses. Let's denote the probability of Matt winning in a given state i as P_i. We know that if Matt has no money, he loses, or that $P_0 = 0$. We know that if Matt has all of the money, he wins, or that $P_7 = 1$. We can also state the probability for a given middle state as $P_i = \frac{1}{2}P_{i-1} + \frac{1}{2}P_{i+1}$. So, the probability that Matt will win in each state can be represented as a system of equations.

$$
\begin{aligned}
P_0 &= 0 \\
P_1 &= \frac{1}{2}P_0 + \frac{1}{2}P_2 \\
P_2 &= \frac{1}{2}P_1 + \frac{1}{2}P_3 \\
P_3 &= \frac{1}{2}P_2 + \frac{1}{2}P_4 \\
P_4 &= \frac{1}{2}P_3 + \frac{1}{2}P_5 \\
P_5 &= \frac{1}{2}P_4 + \frac{1}{2}P_6 \\
P_6 &= \frac{1}{2}P_5 + \frac{1}{2}P_7 \\
P_7 &= 1
\end{aligned}
$$

Solving this system yields that Matt has a $\frac{3}{7}$ probability of winning.

Problem 0.5

Your first project will be to create a program to solve puzzles of a nature similar to this game: http://www.miniclip.com/games/bloxorz/. The objective of the game is to drop the 1x2x1 block through the hole in the middle of the stage without falling off of the sides. Obstacles, such as bridges triggered by switches, may also lie between you and your goal.

Try solving the first two levels. Did you find a path to the goal in the fewest number of moves? How did you go about finding a path? Did you use a systematic or a more random approach? Please turn in the passcode for level 3.

Solution:

For the path finding section, we were looking for some variation of a depth first search or greedy local search. For example, finding any solution to the goal state and then incrementally trying to improve it. In general we accepted any reasonably defined strategy to find a good solution.

The passcode for the third level is 918660.