Homework 7

More Predicate Logic (and some DT stuff)

Due: 6:00PM on 11/11/09

Problem 7.1

A finite probability space, $(\Omega, P, 2^\Omega)$ [1], where $P : 2^\Omega \rightarrow [0, 1]$ has the following properites:

a. $P(\emptyset) = 0$

b. $P(\Omega) = 1$

c. $P(A \cup B) = P(A) + P(B \backslash A)$

Conditional probability is defined as the probability of A given B and written as $P(A|B) := P(A \cap B)/P(B)$.

a. Let $P_B(A) = P(A|B)$. Show that $(B, P_B, 2^B)$ is a probability space.

b. Show that $P(A|B) = P(B|A) \cdot P(A)/P(B)$.

Problem 7.2

Given the following axioms representing the group laws:

a. $\exists n, \forall a(f(n, a) = a = f(a, n))$

b. $\forall a, b, c(f(f(a, b), c) = f(a, f(b, c)))$

c. $\forall a \exists b(f(a, b) = n = f(b, a))$

Show that the inverse is unique.

Problem 7.3

Unify the following expressions or state why they cannot be unified.

a. $in(X, Y)$ and $in(Z, office - of(Z))$

[1] 2^S where S is a set denotes the power set of S, which is the set of all subsets of S.

b. $in(X, X)$ and $in(Z, office - of(Z))$

c. $p(X, b, b)$ and $p(a, Y, Z)$

d. $p(Y, Y, b)$ and $p(Z, X, Z)$

e. $p(f(X, X), a)$ and $p(f(Y, f(Y, a)), a)$