

CSCI 1590: Homework 3  
Professor John Savage  
**Assigned:** February 25, 2009,  
**Due:** March 9, 2009

1. Show that a circuit for a Boolean function (one output vertex) over the standard basis can be transformed into one that uses negation only on inputs by at most doubling the number of AND, OR, and NOT gates and without changing its depth by more than a constant factor.

**Hint:** Find the two-input gate closest to the output gate that is connected to a NOT gate. Change the circuit to move the NOT gate closer to the inputs.

2. Over the basis  $B_2$  derive good upper and lower bounds on the circuit size of the functions  $f_4^{(n)} : \mathcal{B}^n \mapsto \mathcal{B}$  and  $f_5^{(n)} : \mathcal{B}^n \mapsto \mathcal{B}$  defined as

$$\begin{aligned} f_4^{(n)} &= ((y + 2) \bmod 4) \bmod 2 \\ f_5^{(n)} &= ((y + 2) \bmod 5) \bmod 2 \end{aligned}$$

Here  $y = \sum_{i=1}^n x_i$  and  $\sum$  and  $+$  denote integer addition.