

CS 159: Homework 3
Professor John Savage
Assigned: February 27, 2008,
Due: March 10, 2008

Question 1

In class (lecture 9) we highlighted the connection between NP, PSPACE, puzzles and games. Asking someone to determine if $\mathbf{x} \in SAT$, is similar to asking someone to solve a puzzle. Finding the solution to a puzzle may be challenging, but once solved, its solution is easy to verify.

Determining membership in a PSPACE-complete language, such as TQBF, appears more difficult. Asking whether $\mathbf{x} \in TQBF$ is similar to asking whether a certain game has a winning strategy for one of its players. To see why, consider the following two-player game:

Initially players are presented with the boolean formula, $\phi(\mathbf{x})$. On the first turn, player 1 decides whether variable x_1 is 1 or 0. On the second turn player 2 decides whether variable x_2 is 1 or 0. Players alternate in this fashion until all variables have been assigned values. Player 1 wins if $\phi(\mathbf{x}) = 1$, player 2 wins if $\phi(\mathbf{x}) = 0$.

Now consider the language *BGAME* defined as follows. A boolean formula $\phi(\mathbf{x}) \in BGAME$ if and only if there is a winning strategy for player 1 when $\phi(\mathbf{x})$ is used in the game described above. Here a “winning strategy” means that there is a way for player 1 to ensure victory regardless of the choices made by player 2.

Show that *BGAME* is PSPACE-complete by giving a reduction from *TQBF* to *BGAME*. Note that in an instance of *TQBF* there may be more than one variable per quantifier.

Question 2

1. How would you justify the importance of P and NP? What about diagonalization?
2. Of all the proofs we have given, which did you find the most difficult? Please provide a few questions that indicate which part of these proofs you feel are challenging.

Even if you are no longer confused by these proofs, you should still provide questions that highlight the aspects of the proof that you feel are most important in understanding what is going on.

3. Based on the results we’ve seen so far, what are some extensions and/or generalizations of these results that you are curious about?