

CS 159: Homework 4
Professor John Savage
Assigned: March 13, 2009,
Due: March 20, 2009

1. An **n -superconcentrator** is a directed acyclic graph $G = (V, E)$ with n input vertices and n output vertices and the property that for any r inputs and any r outputs, $1 \leq r \leq n$, there are r vertex-disjoint paths in G connecting these inputs and outputs. (Paths are **vertex-disjoint** if they have no vertices in common.)
 - (a) Prove that to pebble any $S + 1$ outputs of an n -superconcentrator, $S + 1 \leq n$, from an initial placement of S pebbles requires that at least $n - S$ different inputs be pebbled.

Hint: Suppose that at most $n - (S + 1)$ inputs are pebbled from an initial placement of S pebbles to pebble $S + 1$ outputs. Can you reason from the superconcentration property that $S + 1$ or more inputs cannot remain unpebbled since $S + 1$ outputs are pebbled?
 - (b) Use the result of question (a) to show that to pebble an n -superconcentrator with S pebbles in time T requires S and T to satisfy the following inequality:

$$(S + 1)T \geq \frac{n^2}{2}.$$

Hint: As in the proof of Theorem 10.4.1 divide time up into consecutive intervals. Choose the intervals so that each has the same number of outputs pebbled during it. Apply the results of the previous problem to obtain a lower bound on the sum of the number of input and output vertices that are pebbled during the interval.