Lecture 2 Notes

2 Feb 2003

1 Inverse Function Theorem

We're in the midst of proving the Inverse Function Theorem; we've simplified to the case where we have a function f whose derivative at a point b is not just nonzero, but is actually the identity. So our statement looks like this:

Theorem: Suppose $f : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is a map, and E is a closed set in $\mathbb{R}^n$ containing a point b in its interior $\det f'(b) = I$. Then there exist open sets $V \subset \mathbb{R}^n$ and $W \subset \mathbb{R}^n$ such that

- $b \in V, f(b) \in W$,
- $f|V : V \longrightarrow W$ has a continuous inverse $f^{-1} : W \longrightarrow V$,
- f^{-1} is differentiable at every $y \in W$, and
- $(f^{-1})'(y) = [f'(f^{-1}(y))]^{-1}$ for all $y \in W$

We've proved the existence of a closed rectangle U with several properties:

(1) For $x \in U, \det(f'(x)) \neq 0.$

(2) For $x \in U, \|D_j f^i(x) - D_j f^i(b)\| < \frac{1}{2n^2}.$

(3) For $x \in U, f(x) \neq f(b),$ unless $x = b.$

(4) For $x_1, x_2 \in U, \|x_1 - x_2\| < 2|f(x_1) - f(x_2)\|.$

We'll now proceed to show something about how f maps the boundary of U into $\mathbb{R}^n$. I'll use ∂U to denote the boundary of U, for reasons that'll be clear in about 5 weeks.

What we're going to show is that

(5) For $y \in W$ and $x \in \partial U$, $\|y - f(b)\| < \|y - f(x)\|.$