STEVENS

ACCORDING to the ancient philosopher Heraclitus, the sole actuality of nature resides in change. All things are becoming. All things are flowing. At the same time, however, all things remain the same. The modern physicist wrestles with that paradox when he studies flow and turbulence. The new water chases out the old, but the pattern remains the same.

Turbulence

TURBULENCE forms the primordial pattern, the chaos that was "in the beginning." We are all familiar with turbulence. We have poured cream into coffee and watched the white marbled swirls as they curl and twist. We have watched the smoke from the fire stream upward and break into whirls and eddies. But the exact visual pattern is difficult to describe. Turbulence does not fit precisely any of the simple patterns that we generated in the last chapter. To some extent turbulence resembles the random meanders of Figures 23*b* and 29, and it happens that any particular particle within a turbulent flow does indeed describe an erratic and meandering path. But turbulent flows also have eddies and whirls like the configurations of Figures 23*a* and 30*a*. In fact, it is the eddies that distinguish turbulent from nonturbulent or laminar flow.

We will study spiral patterns more fully in the next chapter, but here let us observe that the spiral eddy comes into existence when a stream gets stalled against its boundaries or against another stream moving in the opposite direction. The stalled stream breaks into pieces that roll over on themselves. Right at the boundary, the flow of the stream has zero velocity — which is why little particles of dust can ride on the blade of a fan without being blown off, and why you cannot blow fine pieces of dust from the surface of a table — only large pieces that stick up into the breeze. At increasing distances from the boundary, the flow moves with higher velocities, and the difference in rates of flow causes the stream to trip over itself, to curl around on itself, just as a wave curls when it stubs its toe rushing up the beach.

Although we expect to find eddies in turbulent flow, we do not know when any specific eddy will come into being or die away. We cannot yet predict how eddies interact. Similarly, we know as a general rule that any particle within a turbulent flow gets knocked about in an aimless fashion by the swirls, so that it describes an erratic meandering path, but at any given moment we cannot predict the precise location or velocity of the particle. Our inability to predict details of turbulent flow hampers us in many fields. We find it difficult to forecast the weather, interpret sunspots, ascertain the flow of material beneath the earth's crust, or even to predict the exact pressure required to force a large volume of water through a long pipe. Much about turbulence, like the spreading of the clouds, remains beyond our understanding.

But even if we cannot predict all the details, we can predict something about the average case. We can consider the unpredictable local velocities and pressures as chance or random occurrences and then, with the aid of probability theory, take the mean of those occurrences and obtain mathematical descriptions of average motions in average flows.

Just because such an analysis treats the eddies as random occurrences, we should not be misled into believing that they really are random. An eddy is determined by other eddies, and those in turn are determined by still others, and so forth, back to certain specific initial conditions. But we cannot yet describe the initial conditions with enough accuracy to be able to predict all the resulting consequences. The initial conditions contain so many factors that compete with and countermand one another that we are forced to treat the resulting eddies as chance events. When we do, we get results that describe the average, that is to say, the most probable case.

The analysis of turbulence in terms of probability reveals several interesting things about eddies. For instance, the average eddy moves a distance about equal to its own diameter before it generates small eddies that move, more often than not, in the opposite direction. Those smaller eddies generate still smaller eddies and the process continues until all the energy dissipates as heat through molecular motion. In 1941, A. N. Kolmogoroff first set forth the idea that turbulence generates a hierarchy of eddies, thereby inspiring the beautifully apt verse of L. F. Richardson:

Big whirls have little whirls,
That feed on their velocity;
And little whirls have lesser whirls,
And so on to viscosity.

Through statistical analysis, Kolmogoroff also predicted that the velocity of an eddy is proportional to the cube root of its size, that, for example, an eddy moving twice as fast as another will usually be eight times as large, or that one moving ten times as fast will be a thousand times as large.

Reynolds Number

STILL ANOTHER WORK of a theoretical and statistical nature, this time by Werner Heisenberg, shows why density, viscosity, and the width of a stream all play a part in the visual appearance of turbulence — just as Osborne Reynolds observed (without being able to explain) about ninety years ago. Reynolds's discovery, as expressed by the concept of the Reynolds number, shows how things can change their shape in response to a change in scale, and yet, at the same time, and in seeming contradiction, have the same shape at different scales. Let us examine that idea.

We can arrive at the concept of the Reynolds number by asking four simple questions: 1, Does turbulence increase or decrease with an increase in the velocity of the stream? 2, Does turbulence increase or decrease with an increase in the size of an obstacle in the stream? 3, Does turbulence increase or decrease with an increase in the density of material that makes up the stream? 4, Does turbulence increase or decrease with an increase in the viscosity of the material of the stream?

The answers, for the most part, are easy. 1, Turbulence increases as the velocity increases. The flag flutters more in the gale than in the zephyr. 2, Turbulence increases as the size of the obstacle increases. The freighter creates more wake than the dinghy. 3, Turbulence increases as the density of the material increases. With greater density, more particles are present to get jostled about: more interaction can, and therefore will, take place. 4, Turbulence decreases with an increase in viscosity. Here we need to know that viscosity is a measure of the internal friction of the stream, the ability of the stream to stick together, to withstand shear. Realizing that air or water with low viscosity is easily made turbulent when it flows around an obstacle, while oil or molasses with high viscosity oozes smoothly around an obstacle without eddies and backwash, we conclude that turbulence is inversely proportional to viscosity: the greater the viscosity the less the turbulence.

We can write the answers to the four questions in mathematical shorthand by saying that turbulence, T, is directly proportional to velocity, obstacle size, and density — V, S, and D — but it is inversely proportional to viscosity, v. Mathematically then:

$$T \propto V$$

$$T \propto S$$

$$T \propto D$$

$$D \propto \frac{1}{v}$$

or, putting all the terms together,

$$T \propto \frac{V \cdot S \cdot D}{v}$$

We can also go a bit further, just as Reynolds did, for, if we choose our dimensional units in the right way, we can get them to cancel so that T becomes a dimensionless number, the so-called Reynolds number R, and we have

$$R = \frac{V \cdot S \cdot D}{v}$$

Now, the beauty of that derivation lies in the fact that flows with the same Reynolds number look much the same, whereas flows with different Reynolds numbers look quite different. We can combine different velocities, obstacle sizes, densities, and viscosities in different ways, but if we get the same Reynolds number, we will get the same general appearance. Thus, for example, whether a fast-flowing stream is obstructed by a pebble or a slow-moving stream by a boulder, the same pattern of backwash is produced. A speck of dust falls through the air with as much difficulty as our bodies might experience in moving through molasses. Those cases of dynamic similarity are of great interest to the engineer who sets up tests of small models in order to predict the behavior of full-scale structures. The engineer plays the variables of velocity, size, density, and viscosity against one another in any number of ways, but if the variables balance out to the same result, to the same Reynolds number, to the same amount of turbulence, then the flows look roughly equivalent.

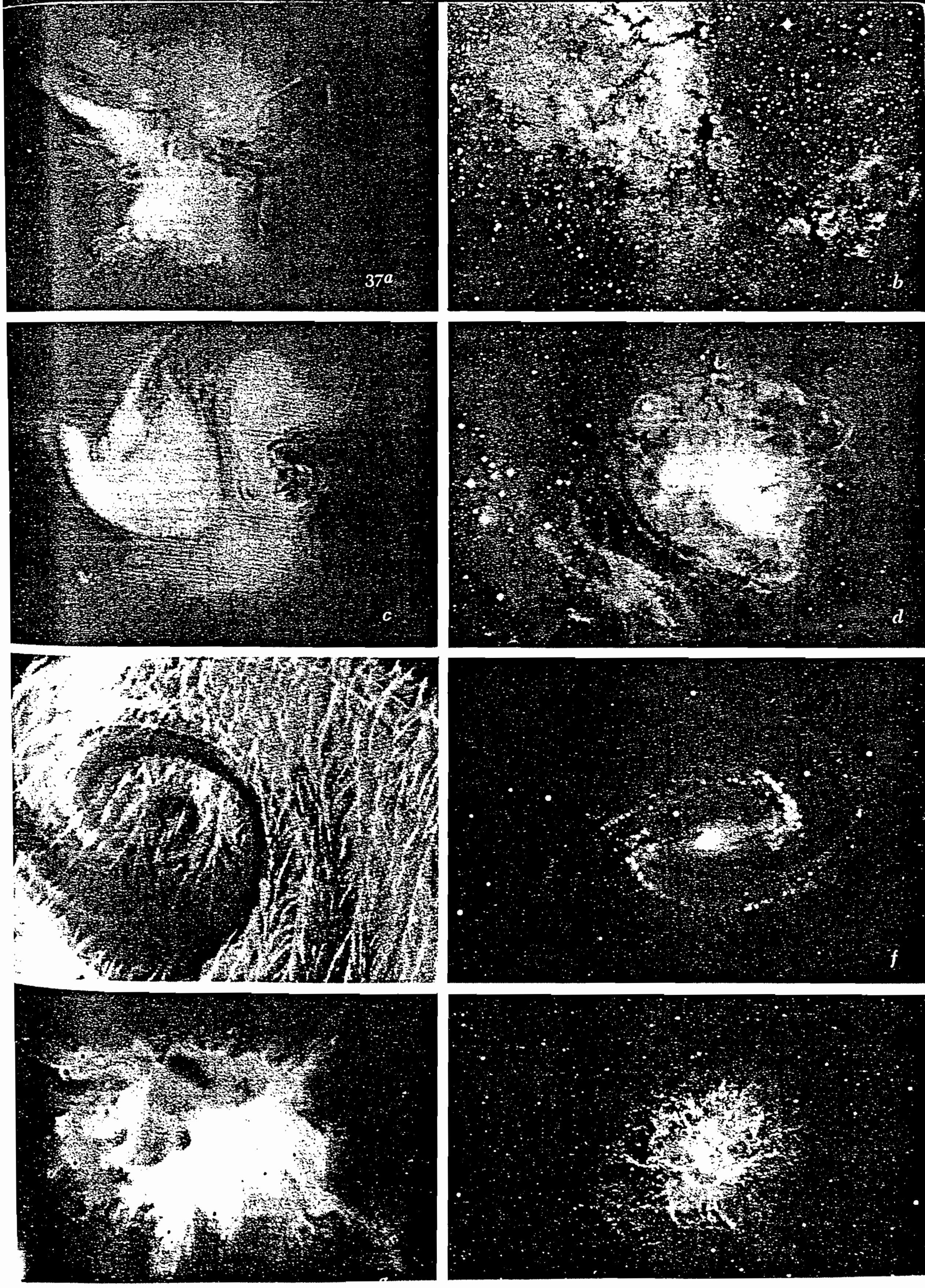
Concentrating on obstacles for a moment, we can see that a change in size results in a change in pattern or form. The small object disturbs the stream but little; the large object creates a turbulent wake. Instead of considering the size of an obstruction, we can also consider the size of a pipe through which the stream might flow. Whereas the large rock directly disrupts the flow, the large pipe allows the flow to disrupt itself. The large pipe has more room for turbulence and thus more turbulence arises. In fact, we can generate Reynolds numbers based on diameters of pipes exactly as we can for numbers based on diameters of obstructions. In both cases the pattern of flow changes with size.

Remembering, however, that decreasing the velocity or density, or increasing the viscosity, can compensate for the effect of increasing size, we see why flows of different sizes can look much the same. Changing only one variable definitely alters the appearance, but changing two or more together may well leave the appearance unaffected. The principle of compensation among variables explains why we find similar patterns at vastly different scales.

One further point should be made about scale and turbulence. Turbulence, or its measure — Reynolds number — is itself an expression of quantity or size. The Reynolds number is a measure of the *amount* of material that is present. Considering flow in a pipe, we can see that increasing the velocity of the flow, the size of the pipe, or the density of the material are simply three different ways to get a greater quantity of material to interact with itself.

The Turbulence of the Universe

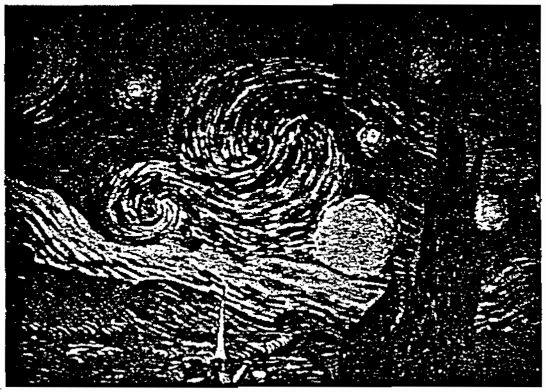
It is no coincidence that milk poured into a wet sink imitates the design of galaxies and clusters of galaxies in the sky (Figure 37). Differences in velocities, densities, and viscosities compensate for the enormous difference in size between the kitchen sink



37a
b
c
d
f

and the heavens, so that the milk and the Milky Way follow a similar plan.

By way of illustration, Figure 37 shows four pairs of pictures. With the exception of *e*, in which the swirling material is a mixture of glycerin, food coloring, and ink, the first frame of each pair shows milk that has been spilled in a black slate sink. The milk covers areas a few inches across, while the gas clouds, spiral galaxies, and exploding Crab Nebula with which the milk is compared cover areas in the order of ten quintillion (10^{19}) miles across.



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Kant and Laplace first described the turbulence of stellar material; Van Gogh's painting, *Starry Night*, gave it visual expression (Figure 38); and Carl von Weizsäcker and George Gamow have attempted to explain the physical facts.

At an early stage, the material of the universe was a gas of nuclear particles. The gas was necessarily turbulent, that is to say, its Reynolds number was necessarily high, because the "pipe" in which it flowed, the absolute size of the universe, was large and did not restrict its flow. The turbulence of the gas gave rise to local compressions and rarefactions. Once the particles of gas were compressed, the gravitational attraction between them increased — their attraction for one another being inversely related as d^2, the square of their separation. It happened that many of the compressed clumps could not expand again: they were held in check by their own gravity. It is interesting to know that similar clumpings of gas take place in the turbulent air around us every day, but that those clumps are too small to hold together under the influence of their own gravitational pull. The clumps of primordial gas, however, were enormously larger. They had a mass several million times the mass of the sun. In such a large clump, the increased gravitational attraction pulled the particles still closer together, increasing the strength of the attraction still more, which in turn pulled the particles closer again. Thus, once a large enough clump had formed, it collapsed on itself in an ever-accelerating gravitational rush.

If the clump of gas was very large, the collapse might continue indefinitely — down to almost nothing. At this very moment there may be billions of such collapsing clumps in the sky. They are the "black holes" that astronomers are looking for, so designated because their immense gravitational attraction makes it impossible for any material to get out, not even particles of light. Of course, they are not really holes. Just the opposite: they are immense concentrations of material that suck everything around them, including light itself, into their interiors. Since no light escapes, we have no way of seeing them directly. Consequently, they have the appearance of black emptinesses.

What happens in the hole, and how matter escapes from becoming infinitely collapsed, remains one of the most pressing questions in physics. What happens in the hole may foretell what will happen to the entire universe, when and if it collapses on itself prior to its next round of expansion. Somehow, according to John Wheeler, material in the hole has the opportunity to take on new spatial properties. Somehow, a new cycle with a new spatial topology starts over again.

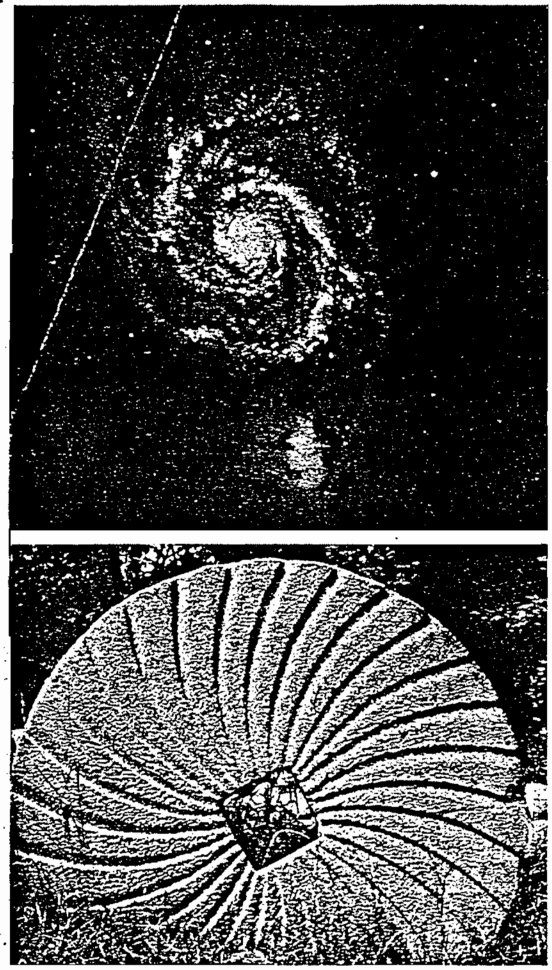
When the clumps of primordial nuclear gas are small, the contraction is stopped by centrifugal force before the stage of the black hole is reached. The random motions within the small clump inevitably cause it to rotate, to behave like an eddy, and the more it shrinks, the faster it spins, like a whirling skater when he pulls in his arms. That increase in rotation leads to an increase in centrifugal force that tends to throw material outward, and soon, in the plane of rotation, material gets flung away from the center in long spiral arms. Perpendicular to the plane of rotation, however, material still moves inward, collapse still goes on, and the whole system flattens into the familiar disk of a spiral galaxy.

The mechanism that keeps the spiral arms of the galaxy spread out, that prevents them from wrapping up, is still not completely understood. As an analogy though, you might consider a rotating water sprinkler that throws out spiral arms of water. Like that sprinkler, a galaxy flings out arms, and the material in the arms feeds into the system from above and below the rotating disk.

Figure 39 shows another analogue of the spiral galaxy, an old millstone that has been cut with spiral tracks that carry ground grain outward from the center as the stone revolves.

When clumps of gas rotate slowly, they do not fling themselves out into spiral arms; they remain smooth and their collapse results in elliptical galaxies.

A similar theory of turbulence and rotation accounts for the formation of the solar system. Laplace assumed that the sun and planets condensed out of a great revolving gaseous cloud. Today we postulate that collections of dust as well as condensations of gas are at work. The particles of dust, incidentally, may have been driven together initially by the pressure of starlight — a force that once again varies inversely with d^2, the square of the separation between particles. According to modern theory, flows of gas and dust break up into turbulent eddies. Those eddies conflict with one another and kill each other off, except for the ones that stay clear of collisions. It happens, not unexpectedly, that the eddies that remain, those that avoid collisions, are spaced at regular intervals from one another. Those eddies condense further and give birth to planets, so that the planets too end up with a regular spacing. The particulars of the theory thus explain why each successive planet in the solar system is about twice as far from the sun as the previous one.



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In that story of the creation of the planets we see a kind of evolutionary theory at work. We see the decimation of the unfit, the swirls that collide with one another, and the survival of the fit, the swirls that were originally positioned so as to avoid collision. The end result is an orderly arrangement that appears more a product of design than chance. Order is born out of chaos. It is interesting, however, that the chaos persists. In fact, considering the dissipated heat generated by the collisions of the eddies, and by the condensation of the dust and gas in forming the planets, the disorder or entropy of the system has actually in-hole in it, the stress trajectories within the plate will look like the lines in the figure. Those lines that run from left to right will be lines of equal tension, and those that run up and down will be lines of equal compression. Where the lines are close together, the forces of tension or compression will be high; and where they are far apart, the forces will be low. If the plate fails, it will tear across the center of the hole just where the lines are tightly bunched.

So why does the same drawing depict those different phenomena?

Richard Feynman has supplied the answer. In discussing the "underlying unity" of nature, he cites the identity between the mathematical description of irrotational flow and the mathematical description of an electric dipole in a uniform field, and he also shows how those phenomena are mathematically equivalent to problems involving the flow of heat, stretched membranes, the diffusion of neutrons, and the uniform lighting of a plane. Then he says:

> The "underlying unity" might mean that everything is made out of the same stuff, and therefore obeys the same equations. That sounds like a good explanation, but let us think. The electrostatic potential, the diffusion of neutrons, heat flow — are we really dealing with the same stuff? Can we really imagine that the electrostatic potential is *physically* identical to the temperature, or to the density of particles? . . . The displacement of a membrane is certainly *not* like a temperature. Why, then, is there "an underlying unity"? . . .
>
> Is it possible that *this* is the clue? That the thing which is common to all the phenomena is the *space*, the framework into which the physics is put? As long as things are reasonably smooth in space, then the important things that will be involved will be the rates of change of quantities with position in space. That is why we always get an equation with a gradient. . . . What is common to all our problems is that they involve space. . . .

Returning then to our figure, we find that each