Why the **Hill** does the hot water always run out?

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Math Modeling, C Block, Teague
12/18/01

Exposition of the Problem

One of the few negative qualities of Hill is the unfortunate condition of its showers. Some showers have such high water pressure that it stings to take a shower in that stall. Meanwhile, others have such low water pressure that it feels like a light rain shower. Assuming that your water pressure is tolerable, the next problem a would-be showerer faces is the scarcity of hot water. Occasionally, there is enough hot water to meet the demand in the morning, but such an occurrence is the exception, rather than the rule.

Fed up with these problems, and given a chance to investigate and model a situation we found interesting, we three, the Hill contingent in our class, decided to investigate this problem. We wanted to quantitatively describe what we had observed over our time showering; to understand and explain what we could now describe; and, if possible, to suggest solutions to these problems. In regards to the latter, we realize that we have essentially no experience with the phenomena we studied and would therefore have trouble offering cogent solutions to the problems we observed; our solution therefore tends more toward finding a model and less toward suggesting solutions to 'fix' the situation.

Assumptions

We made the following assumptions:

- The number of people taking showers in the morning follows a quantized normal distribution over time, $\sigma = 11$ min. = 660 s.
- On an average morning, 35 people each take an 8-minute shower.
- On the individual basis, a person takes a 47°C shower (3/4 the way hot) if the hot water is available; otherwise they take the hottest shower possible.
- The same number of people take showers when the water is running cold as would at the same time if it were running at a normal temperature.
- There is no significant usage of the private showers or any of the sinks in the morning.
- The density of water stays constant for all temperatures, at 1000 g/L.

Analysis of the Problem

Our first step towards solving these problems was to find the plumbing diagrams of the Hill showers. We found the NCSSM plumber, Clifton Gregg, who was kind enough to find the schematics and allow us to study them. We could not photocopy the diagram, but a rough rendition of the significant factors is entered as Appendix 1. We learned that the pairs of showers 2 & 3 and 5 & 6 shared a single .75” pipe, while showers 1 and 4 were each on their own .5” pipe. Showers 1-3 on both floors shared one connection to the main line and showers 4-6 on both floors shared another.

Next, we measured the flow rates of each of the showers. Using a bucket with 1 L graduations, we measured the volume of water passing out of the showerhead in 10s for each shower individually, with three repetitions. We re-measured the flow rates with all the showers on and observed no significant change beyond the bounds of measurement error.

This data was useful in and of itself, since flow rate is related to water pressure, and was also necessary for the later temperature analysis. Our data from this section, as well as a chart of the data, are entered as Appendix 2. We found a great variation in flow rates but also found that flow rate was not the only determining factor in shower comfort. In this regard, the shower head made at least as much of a difference as the flow rate itself. We catalogued the showerheads and found a total of four different heads, of varying size and shape.

Finally, we collected temperature data. To collect this data, we turned all the showers in 1st and 2nd Hill on at the same time, to the maximum temperature and maximum flow. The temperature data was collected once per second using a thermistor, a resistor whose resistance is inversely related to temperature. The temperature was measured until well after the water it had dropped and stabilized at a colder temperature. Data were collected at the shower head, so that there was minimal heat dissipation as the water left the pipe, and after the showers had been unused for some time, to be sure that the hot water supply was not already partially diminished. The thermistor was interfaced to a laptop, and the data collected in Excel. These resistance data were then converted into Celsius using a scale specific to the thermistor and smoothed using a program that removed data points deviating more than .25 °C from the mean of the temperatures within 5 s on either side. A graph of the data is entered as Appendix 3.

Design of the Problem

In designing our model, we first found the combined flow rate of all twelve showers running at once, 2.437 L/s, as well as the average flow rate, .203 L/s.

Next, we numerically integrated, over the applicable time span of 1077 s, the temperature data we had previously collected and subtracted the area (in s•°C) under a line at the equilibrium temperature, 29 °C. The integral gave an untrimmed area of 41803 s•°C and a trimmed area of

10570 s•°C. We thought that the numerical integration was sufficient for our purposes, as the data set seemed acceptably smooth and consistent. A graph showing the numerical integration is entered as Appendix 4. We multiplied (see calculation 1 below) the value of the resulting integral by the aggregate flow rate (in L/s) and the density of water (in g/L). Finally, we multiplied the resulting value (in g•°C) by the specific heat of water, 4.184 J/g•°C. This gave us a value for the overall amount of heat present in the water heater, 107776 kJ, if the energy stored in the water heater at equilibrium temperature were taken to be zero. We called this value the heat reservoir, and we assumed that it would be replenished fully by the water heater given sufficient time.

(1) (41803 s•°C – (29 °C • 1077 s)) • 2.437 L/s • 1.000 g/L • 4.184 J/g•°C = 1.07776 • 10^{8} J

We measured the cold-water temperature to be 17°C. The difference between the equilibrium temperature and this temperature, 12°C, was multiplied by the total flow rate, the density of water, and the specific heat of water, resulting in a power rating for the water heater, 122.4 kW. This tells us that the reservoir, if completely depleted, would take about 15 minutes to fully replenish, with no showers on.

(2) (29 °C – 17 °C) • 2.437 L/s • 1.000 g/L • 4.184 J/g•°C = 1.224 • 10^{5} J/s = 1.224 • 10^{5} W

(3) 1.07776 • 10^{5} kJ / 122.4 kJ/s = 880.4 s = 14.67 min

Modeling the behavior of the shower system in closed form would be difficult, as the shower temperature could not be described consistently under all conditions. Therefore, we chose instead to write a program to step through the process. Each iteration in the program follows the same pattern:

1. Find the temperature at which all showers would be taken, given the current reservoir.
2. Find the number of people taking showers at that time, using a quantized bell curve.
3. Subtract from the reservoir the total energy used by the showers.
4. Add to the reservoir the energy transferred by the heater to the reservoir; this is a constant, 122.4 kJ (since each iteration is 1 s). If this exceeds the heat capacity of the reservoir, set the reservoir to its maximum value.
5. Output the current time, reservoir, shower usage, and shower temperature, into a data file.

The text of the program is entered as Appendix 5.

Solution

The output of this program, as well as an explanation of the results, is entered as Appendix 6.

There are several possible ways to solve the problems:

First, the power input of the water heater could be raised. If it were practical to do this, there could theoretically be a situation at which the temperature bottomed out at or above the comfortable temperature. Perfectly unlimited hot water would require an unreasonable increase in power.

Second, the temperature of the hot water in the water heater could be raised. We are not sure of the exact effect this would have on the situation, since the heat in the reservoir would also change. Regardless, there would be more heat available at the beginning of the simulation, and a given decrease in temperature would not have as great an effect.

Finally, the temperature at which showers are taken could be lowered, or the overall shower usage could be reduced. This introduces an element of sociology into the model, and is together an unsatisfactory solution. Theoretically, were the demand on the system reduced, either by reducing the cost of a single shower or by reducing the total number of showers, the hot water would not decrease as severely or as soon.

In regard to pressure, the showerheads could be standardized to a single suggested head size, away from the extremes.

Justification of Model

The results given by the model fit our observations. For example:

The power output of the heater gives us a maximum sustainable shower load of five showers at one time. That is, five showers can run indefinitely at the ideal temperature. This agrees with observations such as after IMs, when only one floor is using the showers. When this happens, the showers can be used extensively without running out of hot water.

Assuming a peak usage at 7:20, our model gives the first shower of the morning being started at 6:56 and the last shower ending at 7:44. This is very similar to observed usage patterns. Again with peak usage at 7:20, our model predicts that hot water first falls below the ideal temperature at 7:15:22, and bottoms out completely at 7:31:17. This is consistent with our own experience.

Our quantized bell curve has a maximum usage of 10 showers. This is realistic, as each floor only has five comfortable showers that are regularly used.

Strengths and Weaknesses

As discussed earlier, our model reflects our observations. Most of our assumptions were kept separate in the program, allowing them to easily be changed if new data were obtained. The program is easily scalable, and could be modified if additional complexities were added.

Unfortunately, we did oversimplify the shower usage patterns. In reality, usage is much more random than a normal distribution, and shower lengths are not consistent. Our choice of a standard deviation was arbitrary, representing scattered qualitative observations of shower usage. Also, our model only takes into account average values; in reality, on different days we would have lesser or greater usage of certain showers and of the showers as a whole.

Contributions

Jadrian – helped collect data, helped design and implement the calculations for model, figured out plumbing diagrams, helped write the presentation/report.

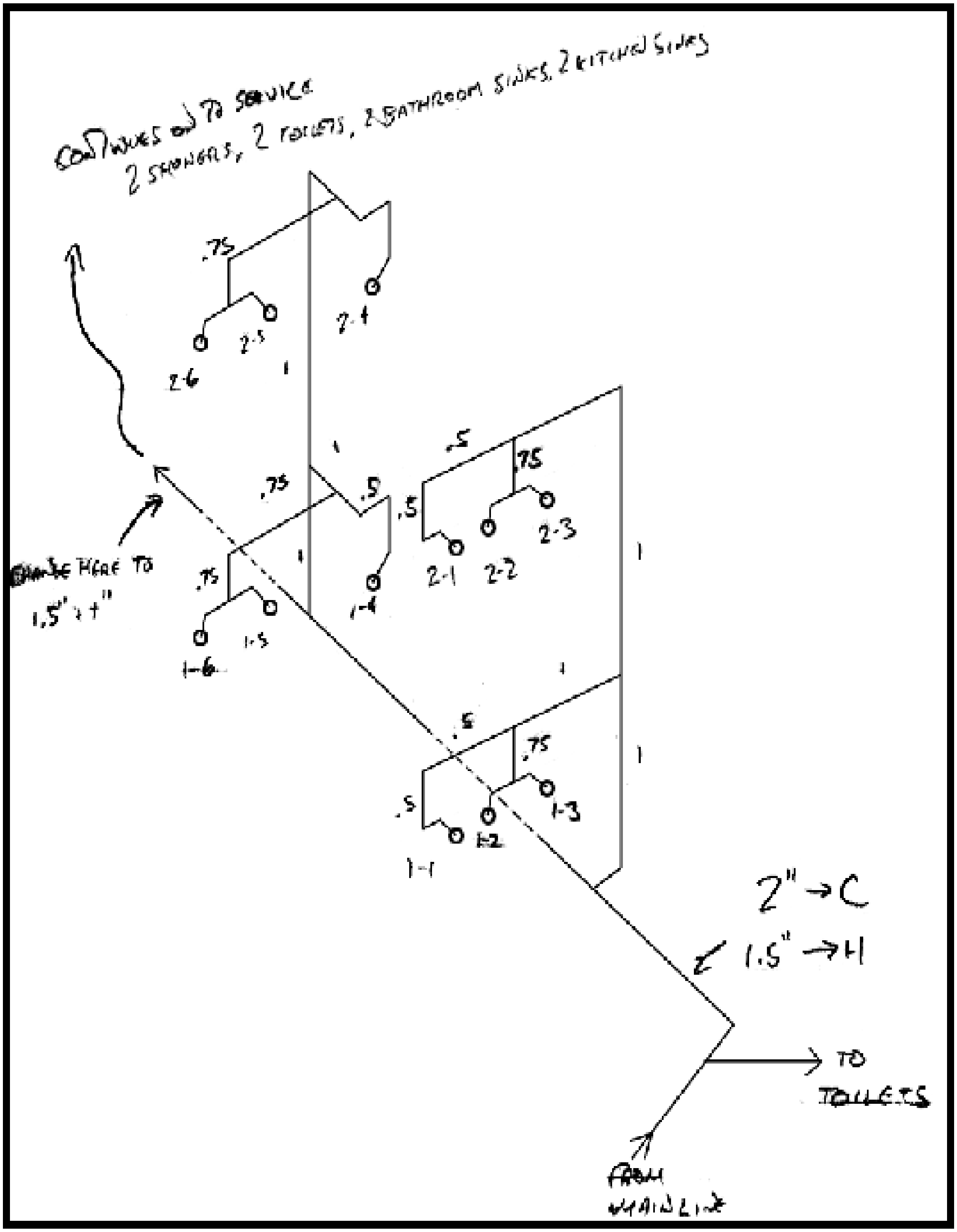
Josh – co-wrote modeling program, figured how to model the system, helped write the presentation/report.

Alex – wrote smoothing program, co-wrote modeling program, assisted in data collection, built the temperature collection system, helped write the presentation/report.

References

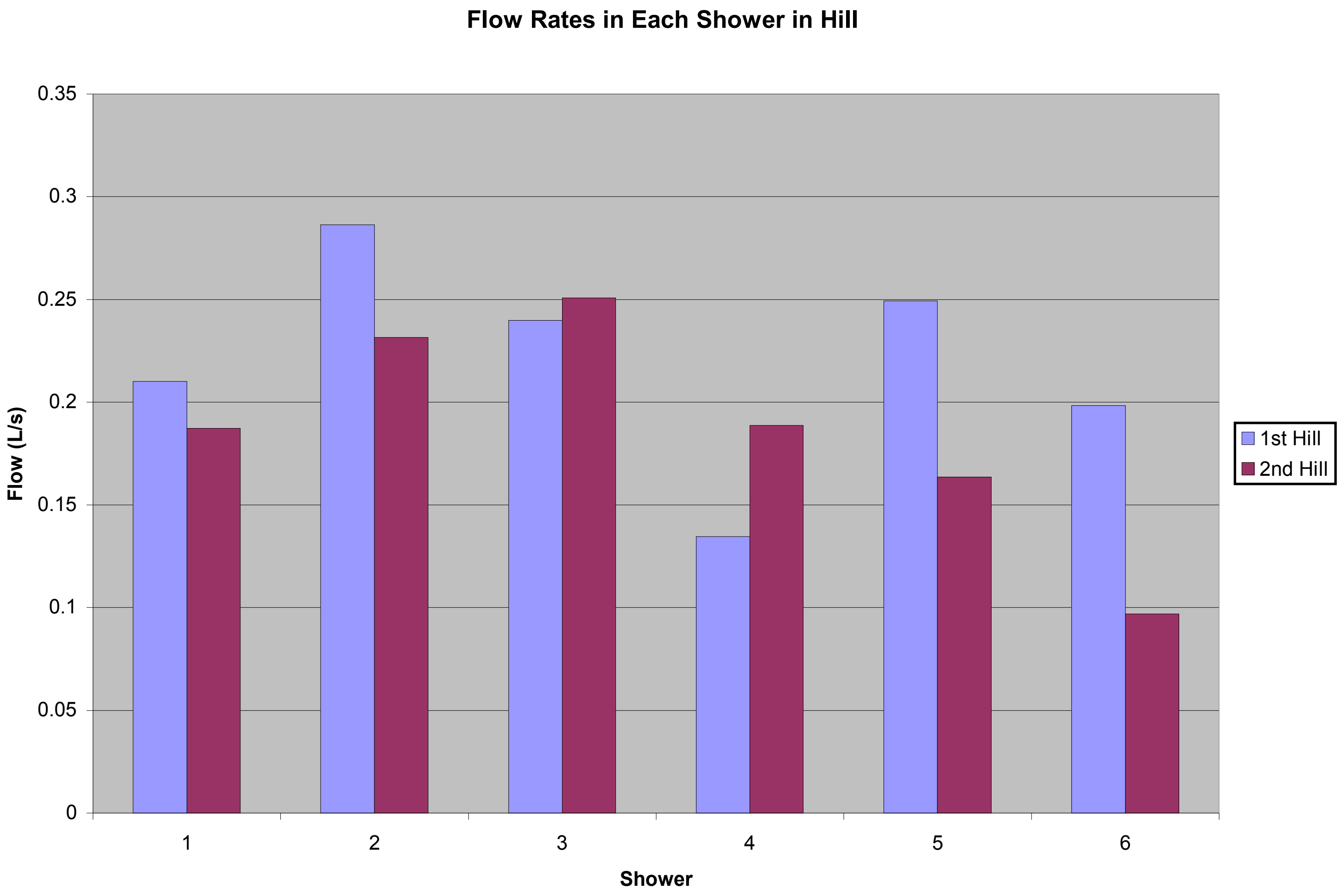
Mr. Clifton Gregg, the NCSSM Plant Facilities plumber, retrieved the plumbing diagrams for Hill that we originally examined.

Appendix 1: The plumbing schematic for the Hill showers



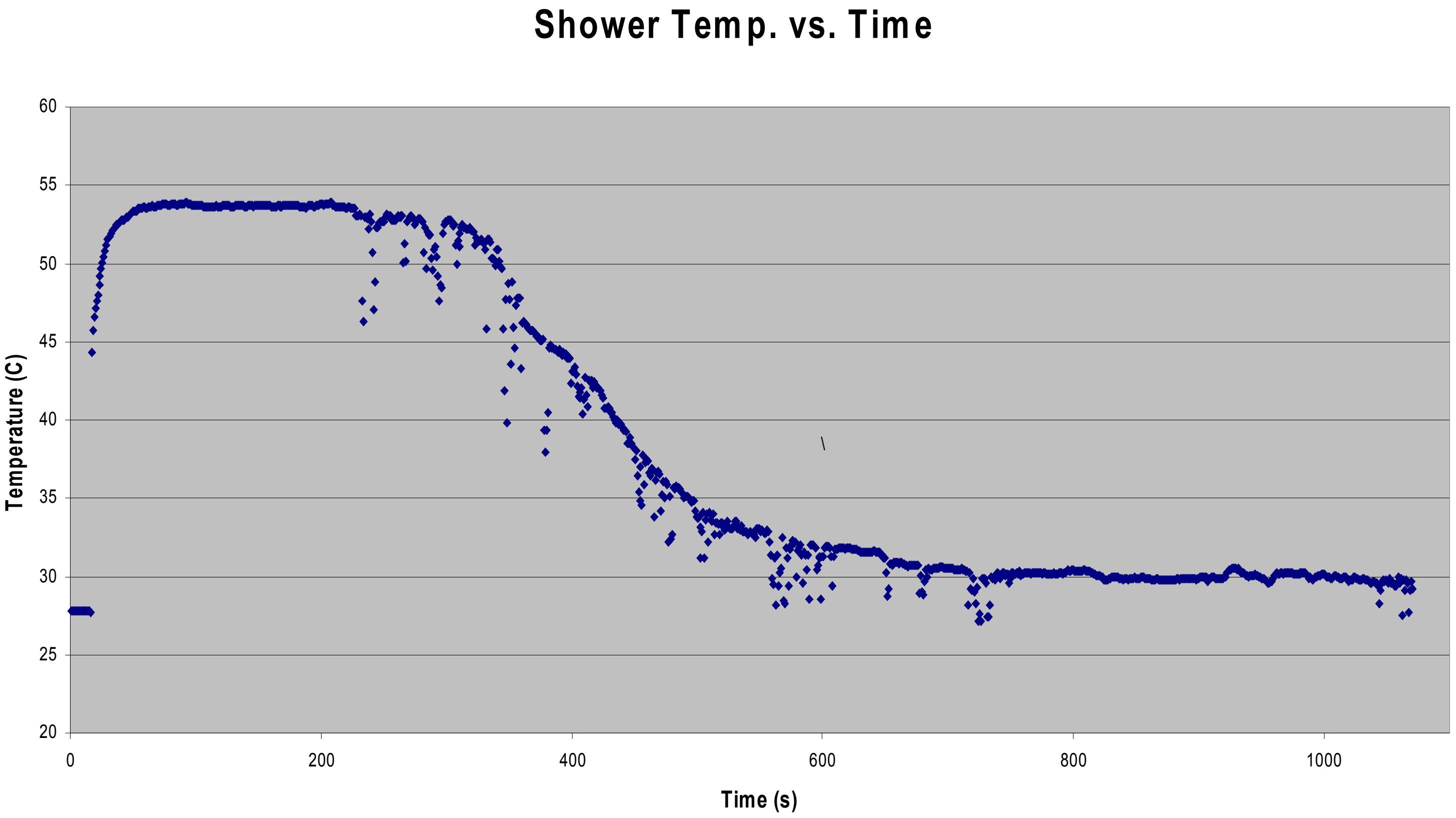
Appendix 2: The Flow Rate Data

Floor	Shower	Test	Volume	Time	Flow	Avg. Flow	Total Flow
1	1	1	2.2	10.42	0.211132	0.210081	2.437134
1	1	2	2.2	10.37	0.21215		
1	1	3	2.2	10.63	0.206961		
1	2	1	3	10.48	0.28626	0.286304	
1	2	2	3	10.32	0.290698		
1	2	3	3	10.64	0.281955		
1	3	1	2.6	10.24	0.253906	0.239853	
1	3	2	2.4	10.17	0.235988		
1	3	3	2.4	10.45	0.229665		
1	4	1	1.4	10.28	0.136187	0.134553	
1	4	2	1.4	10.6	0.132075		
1	4	3	1.4	10.34	0.135397		
1	5	1	2.6	10.31	0.252182	0.249404	
1	5	2	2.6	10.35	0.251208		
1	5	3	2.6	10.62	0.244821		
1	6	1	2.2	10.47	0.210124	0.198445	
1	6	2	2	10.49	0.190658		
1	6	3	2	10.28	0.194553		
2	1	1	2	10.86	0.184162	0.1873	
2	1	2	2	10.84	0.184502		
2	1	3	2	10.35	0.193237		
2	2	1	2.4	10.8	0.222222	0.231473	
2	2	2	2.6	10.73	0.242311		
2	2	3	2.4	10.44	0.229885		
2	3	1	2.6	10.46	0.248566	0.250698	
2	3	2	2.6	10.05	0.258706		
2	3	3	2.6	10.62	0.244821		
2	4	1	2	10.73	0.186393	0.188724	
2	4	2	2	10.7	0.186916		
2	4	3	2	10.37	0.192864		
2	5	1	1.8	10.37	0.173578	0.163455	
2	5	2	1.6	10.28	0.155642		
2	5	3	1.8	11.17	0.161146		
2	6	1	1	10.24	0.097656	0.096843	
2	6	2	1	10.44	0.095785		
2	6	3	1	10.3	0.097087		



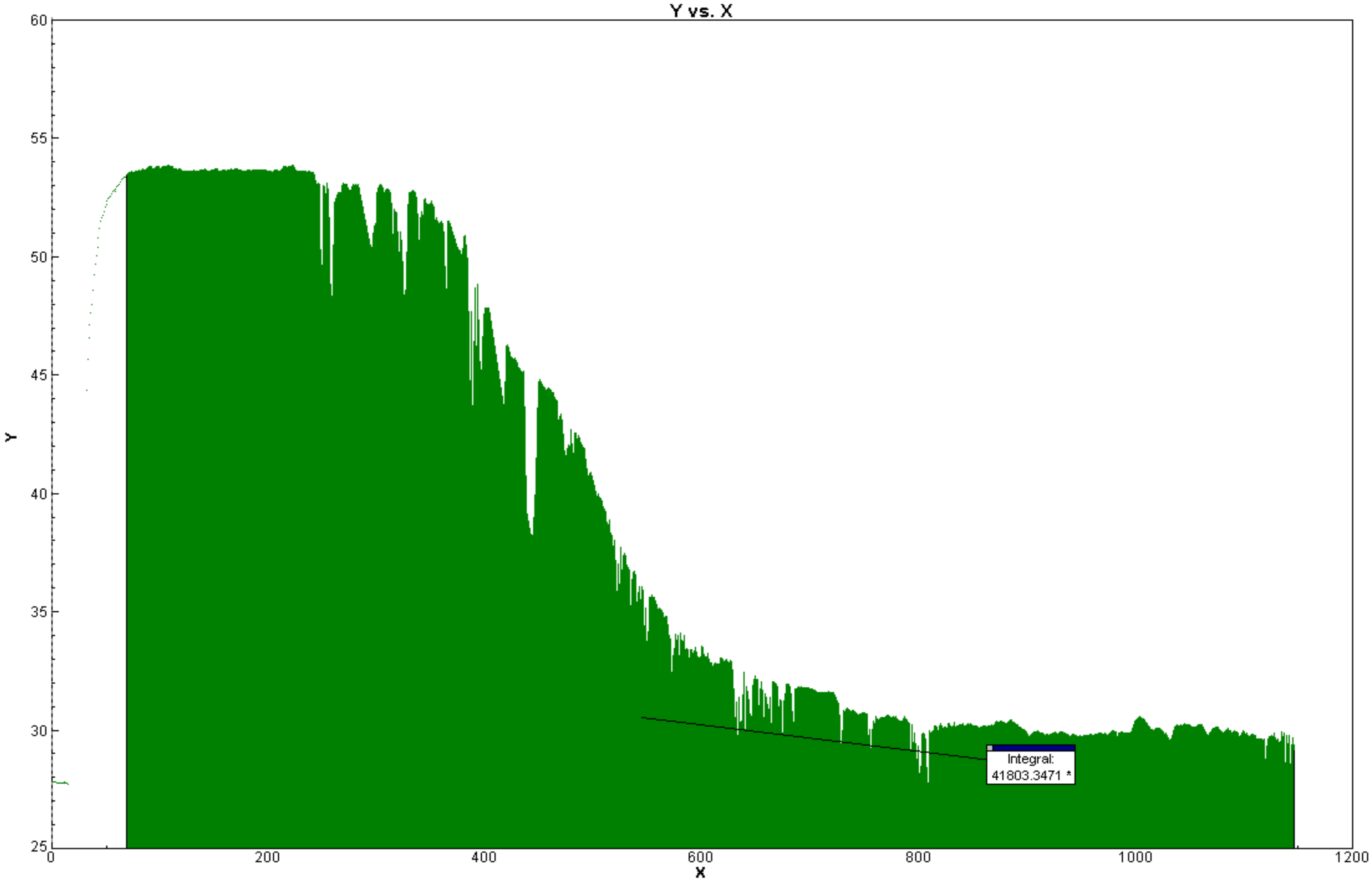
In general, 1st Hill has higher flow rates than 2nd, and earlier showers have a higher flow rate than later, though there are exceptions in both cases.

Appendix 3: Shower Temperature Data



This graph represents temperature data taken from shower 1 on 2nd Hill. The data were smoothed using a program, which removed data points that deviated more than .25 °C from the mean of the temperatures within 5 s on either side of the data point. From this graph we found the maximum temperature of hot water to be 53.5 °C and the sustainable equilibrium temperature to be 29 °C.

Appendix 4: Heat Calculation



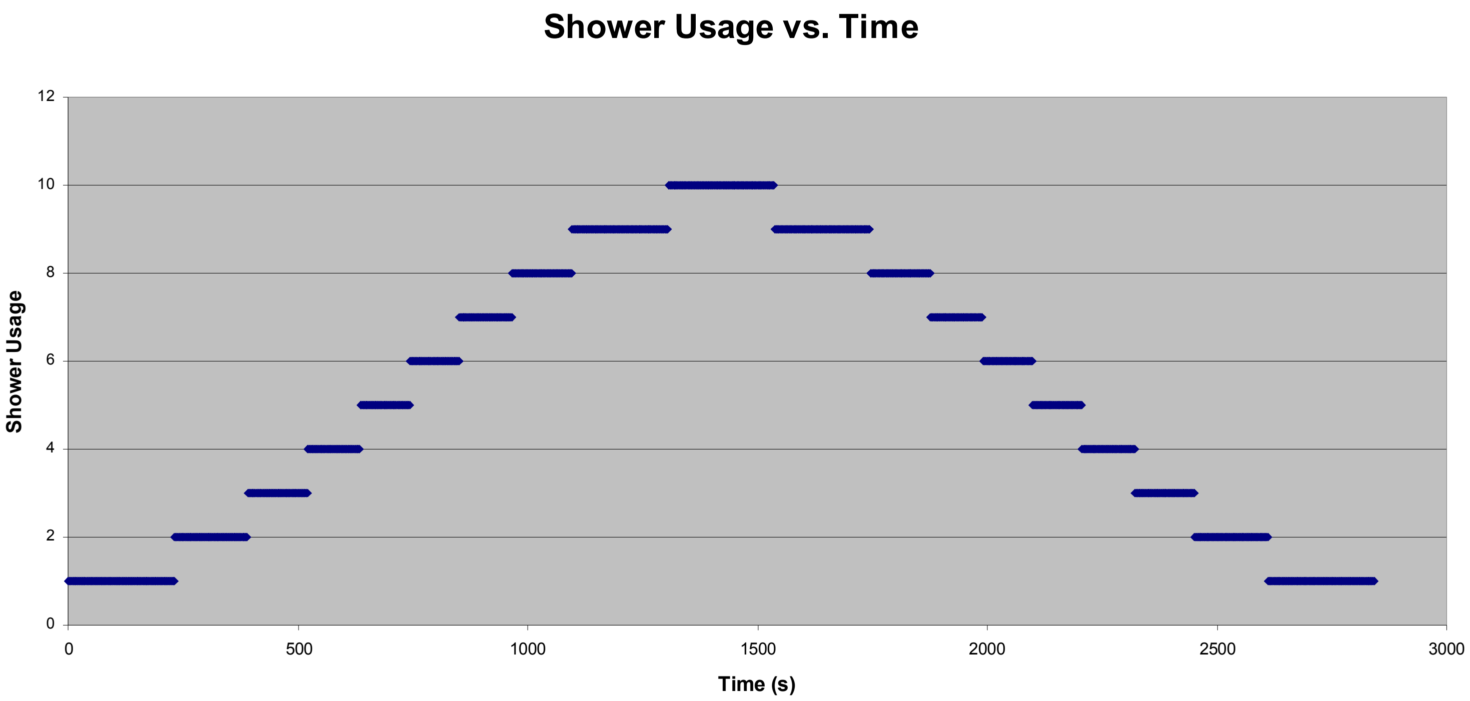
Numerically integrating our data in Graphical Analysis gave a total area under the curve of 41803 °C•s.

Appendix 5: Text of the modeling program

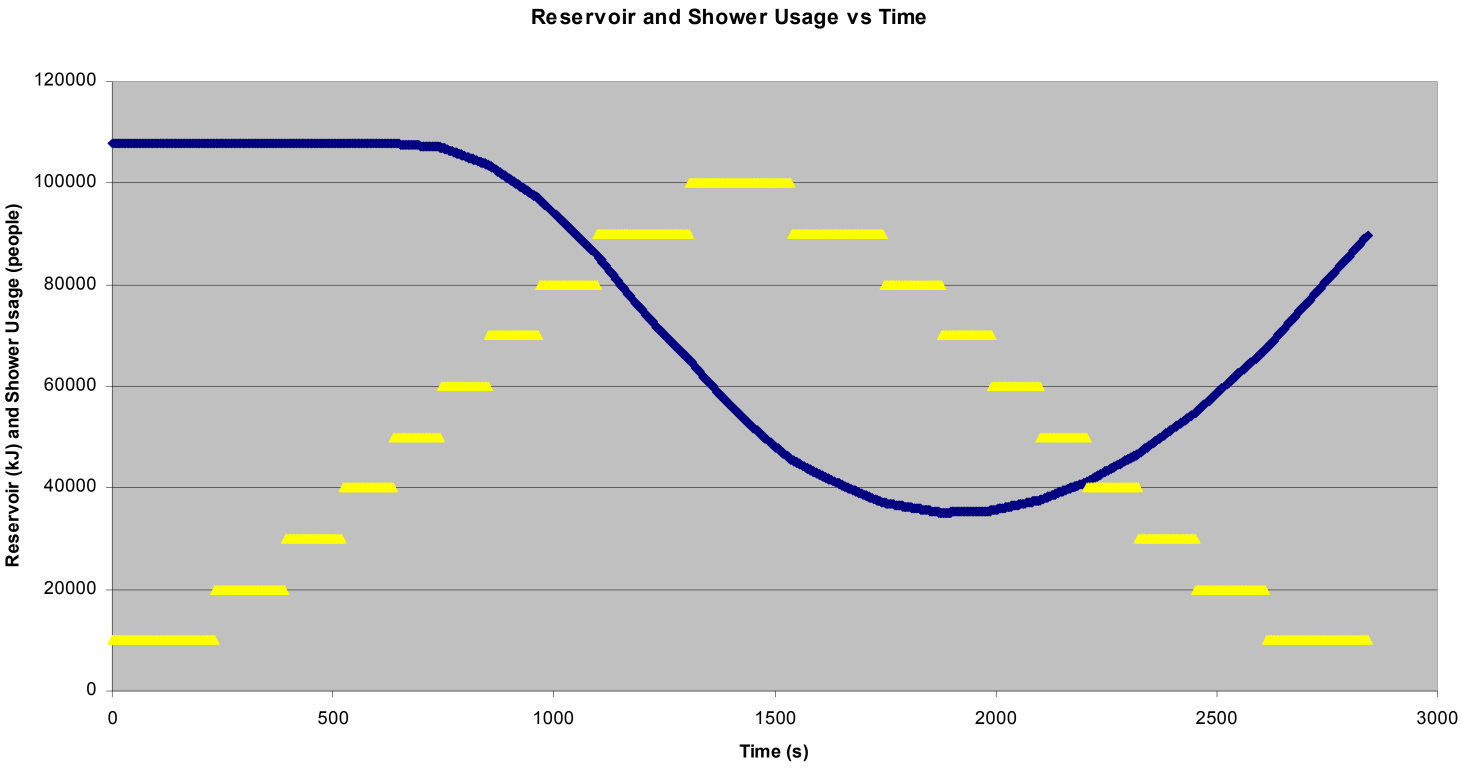
```
#include<iostream.h>
#include<fstream.h>
#include<math.h>
#include<conio.h>
const double initRes=107776;        //kJ
const double heaterPower=122.4;     //kW
const double goodTemp=47.375;
const double equilibrium=29;
const double flowRate=203;          //grams per sec
const double specHeat=.004184;      //Kj per g C
const double mean=1421;
const double stdDev=660;
const double hotTemp=53.5;
const double numPeople=35;
const double avgShow=480;           //sec
const double minEnergy=(goodTemp-equilibrium)*initRes/(hotTemp-equilibrium);

//quantized bell curve of shower use (people) vs time
int showerUse(int time);

//returns temperature people would choose to take their showers at
double temp(double reservoir);

//returns reservoir energy lost through the showers minus the
//energy added by the heater
double energy(double temp, int time);

void main()
{
      clrscr();
      ofstream out;
      out.open("c:\\mydocu~1\\reservoir.dat");
      int time;
      double reservoir=initRes;
      for(time=0;time<=mean*2;time++)
      {
            //subtracts reservoir energy lost through showers
            reservoir-=energy(temp(reservoir),time);
            //adds reservoir energy added from the heater
            reservoir+=heaterPower;
            //the reservoir has a maximum energy
            //any excess energy is dissipated
            if(reservoir>initRes)
                  reservoir=initRes;
            //writes the data to a file readable by excel
            out<<time<<" "<<reservoir<<" "<<showerUse(time)<<" "
                  <<temp(reservoir)<<endl;
      }
      out.close();
}


int showerUse(int time)
{
      double people=(numPeople*avgShow/(stdDev*sqrt(2*M_PI)));
      people*=exp(-pow((time-mean),2)/(2*pow(stdDev,2)));
      return people;
}
```

```
double temp(double reservoir)
{
      double myTemp;
      myTemp=reservoir/initRes;
      myTemp*=hotTemp-equilibrium;
      myTemp+=equilibrium;
      if(myTemp>goodTemp)
            return goodTemp;
      else
            return myTemp;
}

double energy(double temp, int time)
{
      double myEnergy;
      myEnergy=showerUse(time)*flowRate*specHeat*(temp-17);
      return myEnergy;
}
```

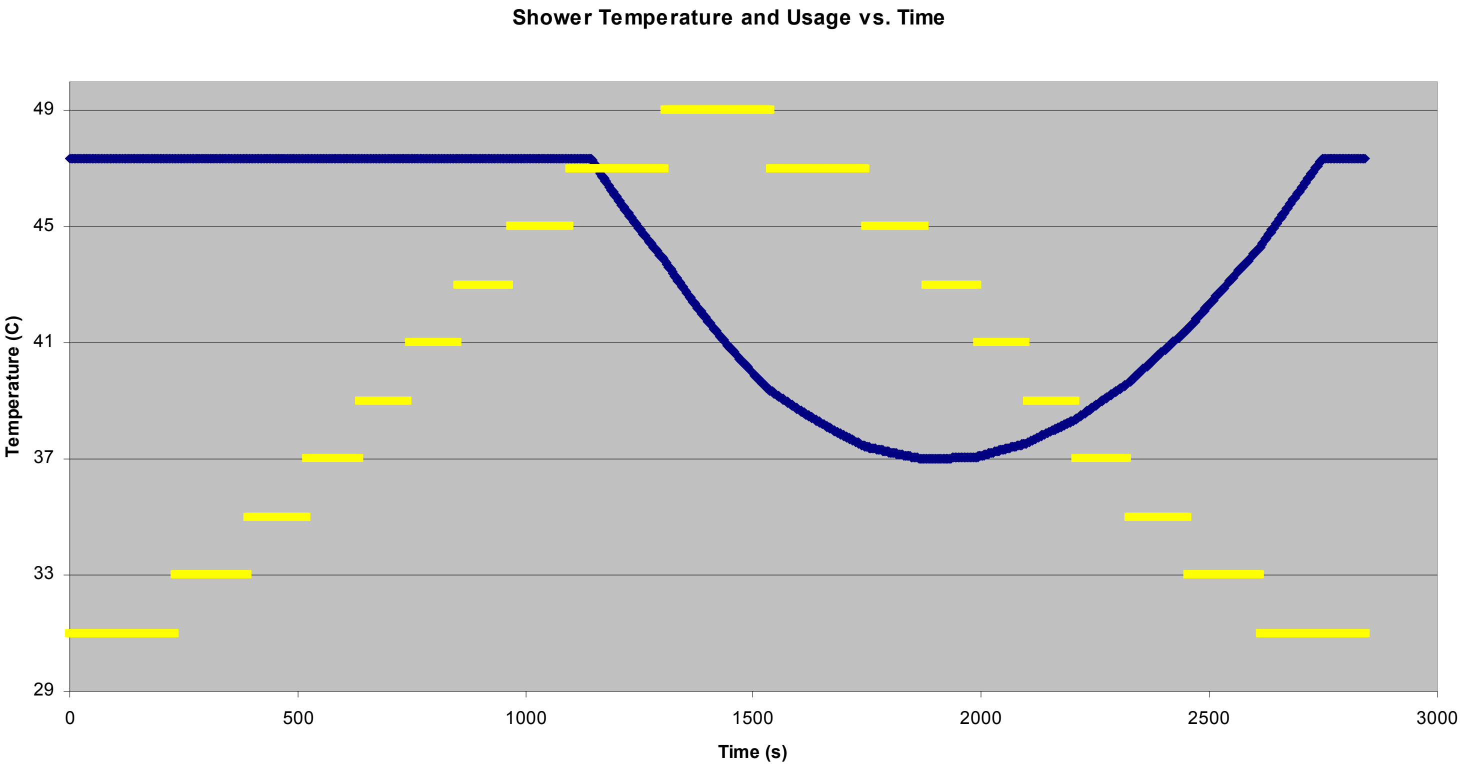
Appendix 6: Application of the Model



The quantized normal distribution used in the program. To create it, we multiplied the values of a normal distribution with σ = 660 by the total number of seconds of shower usage (35 showers • 480 s/shower), and then truncated the decimal portion.



The reservoir energy as a function of time. The decrease starts well before the maximum usage and bottoms out well after. At its lowest value, the reservoir holds roughly one third of its maximum capacity. The energy stored begins to increase even though seven showers are being used, since these showers are colder than ideal.



The shower temperature used as a function of time. It stays constant as long as the hot water is hotter than the ideal shower temperature, then drops as the temperature of the hot water does. The effects of the decreasing reservoir are not felt for some time, as even with a slightly diminished reservoir the temperature may still be hot enough.