Math 196S - Learning and Population Lab

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Abstract

We present a mathematical model that examines the growth of two competing grammars in society, subject to tradeoffs between pressures to communicate and imperfect learning. The preliminary version of the model is a probabalistic description of competition between the grammars in a homogeneous, isolated, and geographically concentrated population. Further revisions of the model add differentiation of age classes, individual mating preferences, and social and familial pressures on grammar acquisition in children.

Building on this local model, we develop a global model that aggregates automaton-like population centers, or "neighborhoods", that follow the local behavior into a larger system. This level of the model allows migration among the neighborhoods, giving a description of large-scale, regional evolution of grammar use in the society. The global model is visualized using a graphical user interface designed for real-time visualization of the model's progress. This simulation allows global results of language use, distribution, and change to be interpreted qualitativly with special attention payed to equilibria and long term regional patterns.

1 Introduction

In this lab, we model the linguistic evolution through time of a society in which two grammars are used to communicate. The two fundamental assumptions that motivate our model are those of imperfect learning and the pressure to communicate: children tend to learn the grammar of their parents, but occasionally pick up the wrong one, and adults are more successful in mating with other adults who use their same grammar.

We expand upon these assumptions below and design a relatively simple but realistic model that represents a large-scale society divided into two age classes and among many geographically self-contained regions called "neighborhoods" that make up a metropolitan area. The model incorporates ecological and social pressures to determine neighborhood-scale, local behavior, and spatial relationships among the neighborhoods as well as local dynamics in each to determine global behavior.

2 Assumptions and Interpretation of the Problem

2.1 The Local Situation

1. Everyone speaks one and only one of grammar A or grammar B.
2. The grammars carry no social associations. They are perfectly identical with respect to intrinsic properties; thus the behavior of the groups speaking each grammar is symmetrical. The grammars' relative value and influence on reproduction and grammar acquisition are determined entirely by the frequency of their use in society.
3. Society can be broken into two major age classes, children and adults, each approximately 20 years in duration. It takes two adults to reproduce, at most one child is produced per mating, and an average on the order of 3 children are produced per pair of adults over their reproductive span.
4. Each individual chooses a grammar based on various pressures during childhood, and continues to use that grammar for the rest of her life.
5. Parents who use the same grammar exert a special influence on grammar choice in their children, but those who use different grammars "cancel each other out", as a consequence of the intrinsic equality of the grammars assumed above.
6. Social pressures, especially from other children, decide each child's grammar choice to the extent that it is left undetermined by his parents.

7. The reproductive success of a given adult is affected by her ability to communicate with the rest of the population. It is possible, though unlikely, for adults who use different grammars to mate.
8. In a closed society confined to a small geographic area, individuals may be considered homogeneously spatially distributed.

2.2 The Global Situation

9. The population is divided among a number of distinct "neighborhoods", each of which behaves according to the local assumptions above. Each neighborhood has finite resources and can only support populations smaller than some fixed size. We assume the increased death rate implicit in this population cap applies to all groups equally.
10. Migration among neighborhoods is possible but relatively resisted by individuals.
11. An individual's decision to emigrate is based on unsatisfactory conditions for successful reproduction in his current neighborhood and an expectation of better conditions in the destination neighborhood. Nearby neighborhoods are more preferable for emigration than far away ones.
12. Migration decisions are made entirely on an individual basis; neighborhoods may not restrict immigrants from joining the local population.

2.3 The Model

13. No special relationships exist between individuals other than parents and children. One notable consequence of this is that the identities of mates are not tracked.
14. In large populations, it is acceptable to model probabilistic population dynamics with expected values. That is, rather than using random variables, we simply deal with fractional people: if three children are born, and 50% of them are expected to use each grammar, then 1.5 children are added to the populations of users of each grammar. Likewise, if two pools of potential mates are available to a given adult, he will mate with all of them proportionally, and produce fractional children from each mating.
15. The number of neighborhoods is fixed.

3 Development of the Model

We first introduce some terminology and notation that will be used throughout the paper. We use **A** and **B** to indicate individuals or groups that use the respective grammar; that is, to say that a population is 60% **A** means that 60% of the population uses grammar A. We also use the terms **P** and **Q** to refer non-specifically to a grammar and its opposite grammar; that is, if a particular concept relating **P** and **Q** is applied in a situation where **P** refers to **A**, then **Q** refers to **B**. See the table below for a complete reference of symbols and variables used in the model.

Table 1: Symbols and Variables Used in This Paper

Symbol	**Definition**
i	Population class index; $i = 1$ for children and $i = 2$ for adults. We also use $i = T$ to designate values that apply to the two age groups combined.
t	Time, measured in years since the start of the model
a_i	#**A** in population i
b_i	#**B** in population i
p_i	#**P** in population i
q_i	#**Q** in population i
T_i	$a_i + b_i = p_i + q_i =$ Total # individuals in population i
α_i	a_i/T_i
β_i	b_i/T_i
ρ_i	p_i/T_i
σ_i	q_i/T_i
$P[x]$	The probability that event x will occur
$\mathbf{P}w\mathbf{Q}$	The event that a **P** is willing to mate with **Q** that he encounters
$\mathbf{P}m\mathbf{Q}$	The event that a **P** mates with a **Q**
$\mathbf{P}m\mathbf{Q}\rightarrow\mathbf{Q}$	The event that a **P** mating with a **Q** yields a child who chooses grammar **Q**
h	Adult **P**s' preference for mating with other **P**s
k	Influence of parents in children's grammar choice
δ_i	Influence of group i in society on children's grammar choice
r	The natural probability of a mating producing a child
K	The environmental population capacity in a neighborhood
m	The migration scaling constant

3.1 Local Grammar Adoption

We begin with a naïve differential equation model inspired by ecological models that ignores age distribution, in which the population is entirely undifferentiated and unbounded, **P**s mate only with other **P**s, and children always use the same grammar as their parents. Because of assumption 8, we also neglect spatial considerations in the probability of two individuals meeting.

$$\begin{aligned} \frac{d}{dt}p_T &\propto P[\textit{a } \boldsymbol{P} \textit{ will encounter another } \boldsymbol{P}] \\ &\propto (P[\textit{a } \boldsymbol{P} \textit{ exists in the population}])^2 \\ &\propto \rho_T^2 \end{aligned} \tag{1}$$

We modify this model by introducing the possibility of **P**s mating with **Q**s, with reproductive rates of s

and s' when mating, respectively, with **P**s and **Q**s.

$$\frac{d}{dt}p_T = s\rho_T^2 + s'\rho_T\sigma_T \tag{2}$$

Now we wish to remove the assumption that children will choose the same grammar as their parents. We introduce several parameters, each of which is in the range [0,1], to describe the outside influences on adults' mate choices and children's grammar choices; these are listed in the table above. Note that $\delta_T = \sum \delta_i = 1$.

Our interpretation of assumption 7, regarding the choice of mating partners by adults, depends on the mate-preference parameter $h \in [-1, 1]$.

$h = 0$	indicates ambivalence about the grammar of a potential mate; if a potential mate is available, the individual should mate with them, regardless of grammar.
$h = 1$	indicates an absoulte preference in **P**s for other **P**s; **P**s refuse to mate with **Q**s.
$h = -1$	indicates an absoulte preference for speakers of the opposite grammar.
$0 < h < 1$	indicates a partial preference in **P**s for other **P**s. In this situation, **P**s mate with **P**s freely, but have a certain reservation about mating with **Q**s.

This leads us

$$\begin{aligned} P[\boldsymbol{P}w\boldsymbol{P}] &= \begin{cases} 1 & \text{if } h \geq 0 \\ 1+h & \text{if } h < 0 \end{cases} \\ P[\boldsymbol{P}w\boldsymbol{Q}] &= \begin{cases} 1-h & \text{if } h \geq 0 \\ 1 & \text{if } h < 0 \end{cases} \end{aligned} \tag{3}$$

One of the fundamental assumptions of our model dictates that **P**s prefer to mate with other **P**s, so we assume $h \geq 0$ for the rest of the development of the model.

Mating success is a function of mutual interest by the two parties and, as in equation 1, the availability of these parties. Since only adults mate, we must now distinguish the age classes with the population class index, i.

$$\begin{aligned} P[\boldsymbol{P}m\boldsymbol{Q}] &= \rho_2 P[\boldsymbol{P}w\boldsymbol{Q}] \cdot \sigma_2 P[\boldsymbol{Q}w\boldsymbol{P}] \\ &= \rho_2\sigma_2(1-h)^2 \\ P[\boldsymbol{P}m\boldsymbol{P}] &= \rho_2^2 \end{aligned} \tag{4}$$

Assumptions 5 and 6, regarding the influence of parents and society on children's grammar choice, motivate the following equations, which now involve the influence parameters k and δ_i.

$$\begin{aligned} P[\boldsymbol{P}m\boldsymbol{P}\to\boldsymbol{P}] &= k + (1-k)\sum\delta_i\rho_i \\ P[\boldsymbol{P}m\boldsymbol{Q}\to\boldsymbol{P}] = P[\boldsymbol{Q}m\boldsymbol{P}\to\boldsymbol{P}] &= \sum\delta_i\rho_i \\ P[\boldsymbol{Q}m\boldsymbol{Q}\to\boldsymbol{P}] &= 1 - P[\boldsymbol{Q}m\boldsymbol{Q}\to\boldsymbol{Q}] \\ &= 1 - [k + (1-k)\sum\delta_i\sigma_i] \\ &= (1-k)(1-\sum\delta_i\sigma_i) \\ &= (1-k)\sum\delta_i\rho_i \end{aligned} \tag{5}$$

Letting $S_P = \sum\delta_i\rho_i$ and incorporating r, the probability of an individual mating event being successful, we compile the equations above into a differential equation describing the growth of the P-speaking child population. For future analysis, it is useful to note that $S_Q = \sum\delta_i\sigma_i = \sum\delta_i - \sum\delta_i\rho_i = 1 - S_P$.

$$\begin{aligned} \frac{d}{dt}p_1 &= \begin{bmatrix} P[\boldsymbol{P}m\boldsymbol{P}]\cdot r\cdot P[\boldsymbol{P}m\boldsymbol{P}\to\boldsymbol{P}]+ \\ P[\boldsymbol{P}m\boldsymbol{Q}]\cdot r\cdot P[\boldsymbol{P}m\boldsymbol{Q}\to\boldsymbol{P}] + P[\boldsymbol{Q}m\boldsymbol{P}]\cdot r\cdot P[\boldsymbol{Q}m\boldsymbol{P}\to\boldsymbol{P}]+ \\ P[\boldsymbol{Q}m\boldsymbol{Q}]\cdot r\cdot P[\boldsymbol{Q}m\boldsymbol{Q}\to\boldsymbol{P}] \end{bmatrix}\cdot T_2 \\ &= [\rho_2^2(k+(1-k)S_P) + 2\rho_2\sigma_2(1-h)^2S_P + \sigma_2^2(1-k)S_P]\cdot rT_2 \equiv B(\rho, T_2) \end{aligned} \tag{6}$$

Note that the growth rate is multiplied by the total adult population. All the mating functions deal with proportional representations of the two grammars in the adult population, so they describe the growth rate of a grammar in the child population normalized to the total adult population; thus it is necessary to multiply by this value to give the absolute growth of the **P**-child population.

3.2 Local Population Dynamics

We now address assumptions 3, 4, and 9, building aging and death into our model.

Since we assume that each age group is approximately 20 years in duration, and that individuals keep the same grammar once they have chosen it in childhood, we arrive at the aging equations:

$$
\begin{aligned}
\frac{d}{dt}p_1 &= \text{children born} - \text{children growing up} \\
&= B(\rho, T_2) - (1/20)p_1 \\
\frac{d}{dt}p_2 &= \text{children growing up} - \text{adults growing too old} \\
&= (1/20)p_1 - (1/20)p_2
\end{aligned}
\tag{7}
$$

In a traditional logistic growth model, the growth rate of a population is proportional to its ratio to the environmental carrying capacity. We wish instead to incorporate a variable death rate that increases in the same fashion as the population reaches carrying capacity, ultimately killing off as many individuals as are born.

We incorporate new births into both grammars and removal of individuals who have aged out of the adult age class to find a growth rate for the entire population. We wish to remove a fraction of this number of individuals from the total population, and then distribute this death toll evenly throughout the population.

$$
\begin{aligned}
growth &= [B(\rho, T_2) + B(\sigma, T_2) - (1/20)T_2] \\
deaths &= \frac{T_T}{K} \cdot growth \\
deathrate &= \frac{deaths}{T_T} = \frac{growth}{K}
\end{aligned}
\tag{8}
$$

This *deathrate* value is the proportion of each population class that must be killed off over every time interval dt. We now include this population-limiting process along with birth and aging from equation 7 to further refine the differential equations:

$$
\begin{aligned}
\frac{d}{dt}p_1 &= B(\rho, T_2) - (deathrate + 1/20) \cdot p_1 \\
&= B(\rho, T_2) - \left(\frac{B(\rho, T_2) + B(\sigma, T_2) - (1/20)T_2}{K} + \frac{1}{20} \right) p_1 \\
\frac{d}{dt}p_2 &= (1/20)p_1 - (deathrate + 1/20) \cdot p_2 \\
&= \frac{1}{20}p_1 - \left(\frac{B(\rho, T_2) + B(\sigma, T_2) - (1/20)T_2}{K} + \frac{1}{20} \right) p_2
\end{aligned}
\tag{9}
$$

3.3 Global Migration

We now expand our model to a macroscopic scale by aggregating several neighborhoods, each of which obeys the local model, into a global structure that allows migration among the neighborhoods due to population pressures. Assumption 11 dictates that, for any given neighborhood, individuals in a group with a sufficiently low reproductive rate will begin emigrating to nearby neighborhoods in which their expected reproductive rate is higher.

Our measure of reproductive success for **P**s in a neighborhood is the net rate of change of **P** children in the population per adult. The rate of emigration of **P** from the original neighborhood to another neighborhood is based on the ratio of the reproductive success for **P**s in the two neighborhoods.

$$\begin{aligned} F_{P,u} &= \text{Reproductive success of } \mathbf{P}\text{s in neighborhood } u \\ &= \text{Net growth of } \mathbf{P} \text{ per adult} \\ &= \left(\frac{d}{dt}p_1\right) \cdot \frac{1}{p_2} \qquad (10) \end{aligned}$$

Letting $\ell(u,v)$ indicate the distance between neighborhoods u and v, the desirability of neighborhood v to **P**s in neighborhood u is given by

$$D_{P,u,v} = exp\left[-\ell(u,v)\frac{F_{P,u}}{F_{P,v}}\right] \qquad (11)$$

This function is designed to give a desirability curve that increases monotonically in $[0,1]$ as the reproductive success of the target neighborhood overtakes that of the source neighborhood. We protect against divide-by-zero errors by defining $D_{P,u,v} = 0$ when $F_{P,v} = 0$.

Since all migration takes place among members of the adult population, the total of **P**s moving to the target neighborhood is based only on the total number of adults. Scaling the desirability by a small constant, m, to ensure slow migrations (as per assumption 10), we arrive at the migration equation. This is added to the equations from the local model to give the aggregate global behavior.

$$\frac{d}{dt}p_{2,u} = m \cdot \sum_v (F_{P,v,u} - F_{P,u,v}) \qquad (12)$$

4 Observations and Analysis

We implemented the model at the local and global scales in computer programs, converting the differential equations into discrete-time difference equations. Local behavior was first simulated and analyzed in Matlab, and then we wrote a Java program to simulate and visualize several neighborhoods at once. In the global simulation, each neighborhood obeys the local model but also exchanges populations through global migration. Each neighborhood's initial population distribution, population capacity, and social parameters are randomly perturbed from the default values, and the neighborhoods are distributed randomly throughout our "world". Please see `http://www.math.duke.edu/j̃adrian/196S/lab2` for source code.

Our analysis at each time and geographical scale generally consists of experimentation with parameters; we isolate the effects of each parameter by changing it while holding all others fixed and observing the resulting behavior of the system. Since there are a number of nonlinear terms in the differential equations and the parameter space is very large, symbolic mathematical analysis of our model is fairly limited.

In this section, we use a set of default values for the model's parameters chosen to accurately reflect a real human society in which the two grammars are very different from each other. These values are listed in the following table.

Table 2: Default Parameter Values

Param	**Value**	**Description**
dt	.25	Discrete time step
h	.8	Adult **P**s preference for mating with other **P**s
k	.7	Parents' influence on children's grammar choice
δ_1	.9	Children's influence on each other's grammar choice
r	.3	Natural mating success rate
K	10^6	Environmental population capacity
N	200	# neighborhoods
m	.00007	Migration scaling factor

4.1 Isolated Neighborhoods: Long Term

4.1.1 Observations

The initial behavior of our local model is very complex and not easily susceptible to mathematical analysis; we leave a description of this case for the next section. After about one life span (40 years), however, α_1 and α_2 begin growing at more or less equal rates and at close values. Under conditions such as these, our model behaves very regularly: **A** dominates the population if $\alpha_T > .5$ and goes extinct if $\alpha_T < .5$. $\alpha_1 = \alpha_2 = \alpha_T = .5$ is an unstable fixed point, and $\alpha_1 = \alpha_2 = \alpha_T = \{0, 1\}$ are stable. See figure 1.

4.1.2 Analysis

Population change between the fixed points is approximately logistic; modifying h, k, or δ_1 changes the growth rate, though extreme values for the parameters can bring out higher-order behavior that makes the growth less regular. In general, k has the largest effect on the shape of the curves; parental influence on

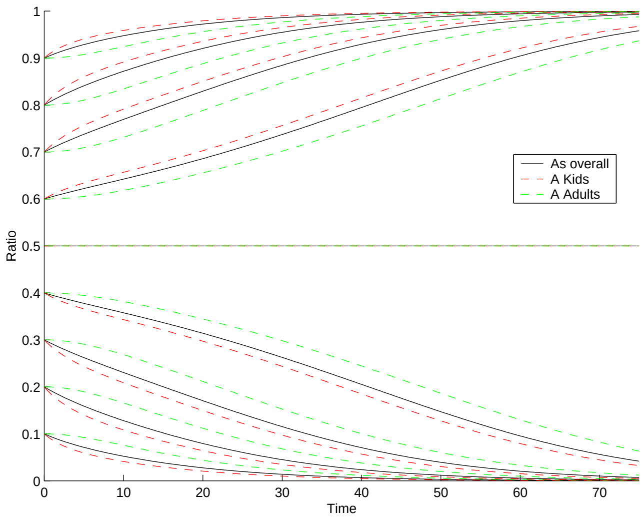


Figure 1: Long-Term Local Behavior with Default Parameters

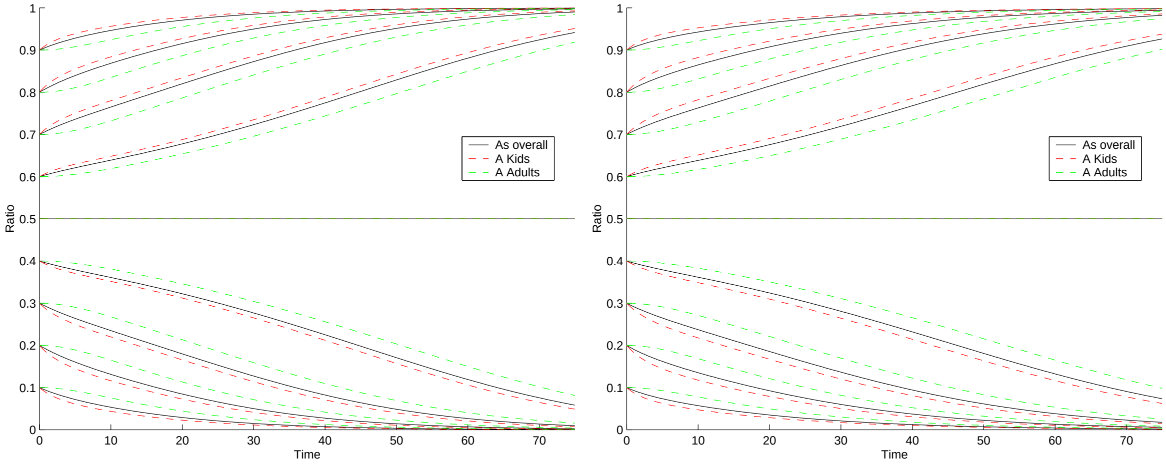
grammar choice creates a feedback loop that drives α_1 toward the extreme value preferred by the adult population. When parents exert no influence on grammar choice, the entire decision is left up to social influences, regardless of parents' mating preferences. When $\alpha_1 = \alpha_2$, this means that both language groups grow at rates exactly proportional to their representation in society, and so the grammar ratio remains fixed. We believe this bifurcation is the only case in which fixed points occur anywhere other than $\alpha_T = 0$, .5, or 1. See figure 2.

It is clear from equation 6 why $\alpha_T = 0$, .5, and 1 are fixed points. If $\alpha_T = 0$, then $B(\alpha, T_2)$ vanishes; likewise, if $\alpha_T = 0$, then $B(\beta, T_2)$ vanishes and $\mathbf{A}$ continues to grow, taking up the entire population. When $\alpha_T = \beta_T = .5$, the B equations are identical and both populations grow at the same rate, maintaining an equal share of the whole population. In general, since a subpopulation's reproductive rate increases as the square of its density, when one grammar gains the slightest majority, its growth rate increases, leading the population into exponential growth. At the same time, the other grammar has a slight disadvantage, and so its relative growth rate declines over time. When population capping comes into effect, the minority group is driven into extinction.

4.2 Isolated Neighborhoods: Short Term

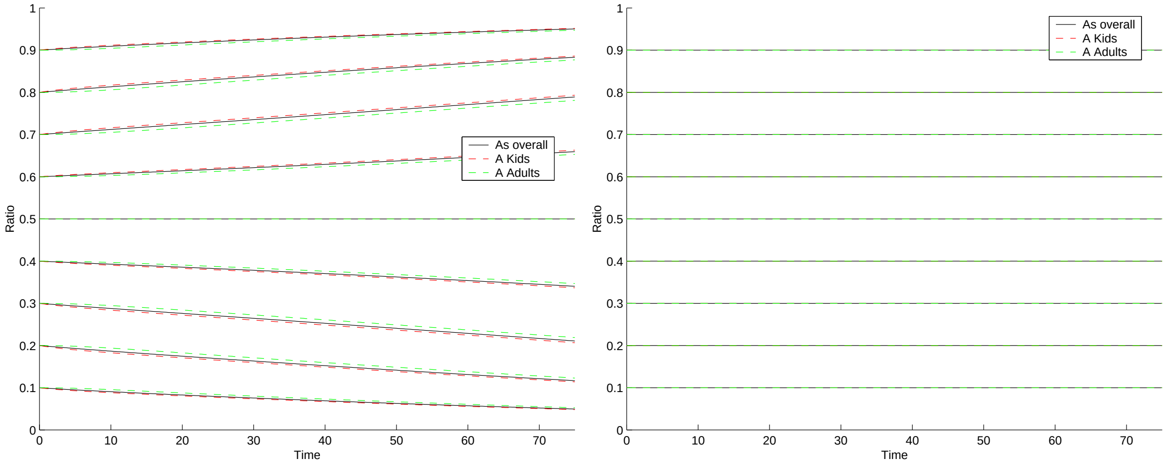
4.2.1 Observations

As mentioned above, short-term behavior in the local model sensitive to nonlinear differential equation; relatively small perturbations in the initial conditions or social parameters can result in the grammar ratio settling on either side of the unstable halfway fixed point.



(a) $h = .1$

(b) $\delta_1 = .1$



(c) $k = .1$

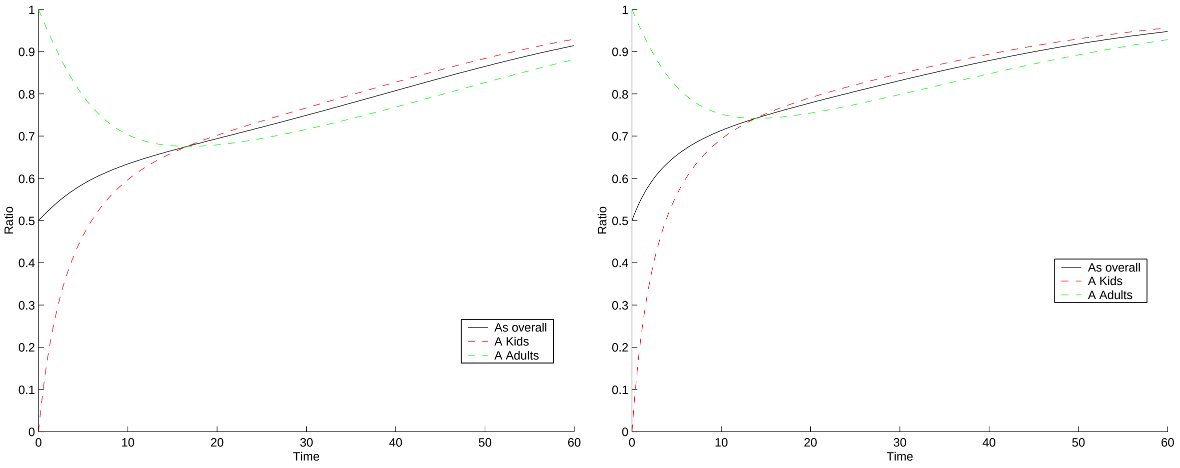
(d) $k = 0$

Figure 2: Long-Term Local Behavior with Modified Parameters

We restrict our discussion to a qualitative analysis of these effects; see figures 3 and 4 for example short-term runs of the local model.

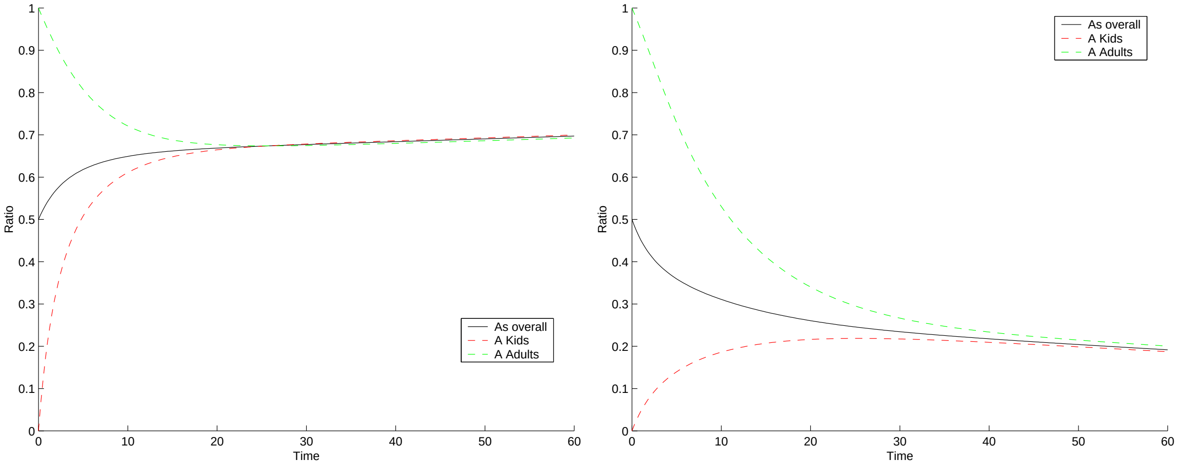
4.2.2 Analysis

In the extreme cases examined here, each age class clearly represents a grammar. Over time, the adults dilute the child population with kids who have picked up their parents' grammar, while the children grow up into adults. This brings the populations toward a grammar balance; modifications of the social paramaters affect the relative rates of this convergence, thus determining the state at which the population enters the



(a) Default Parameters

(b) $\delta_1 = .1$

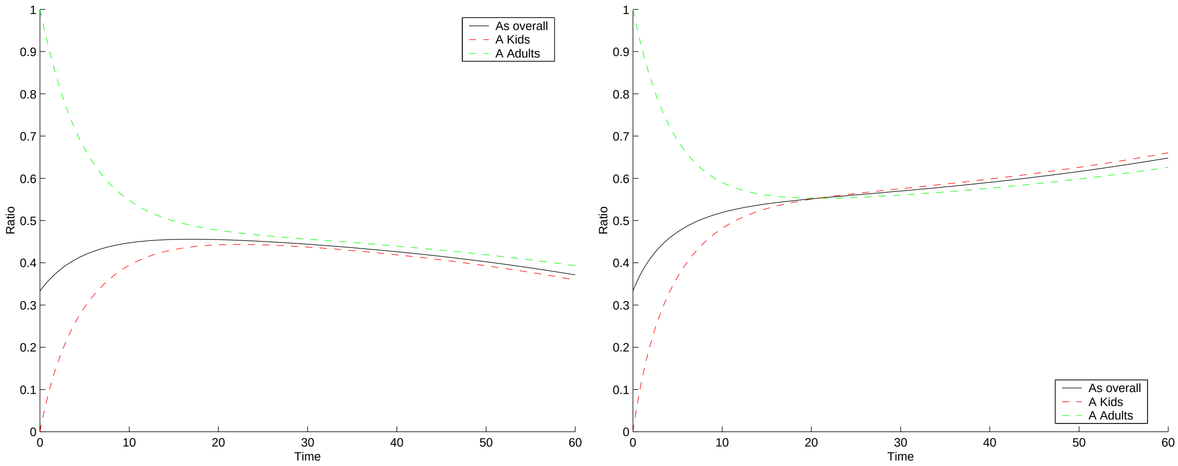


(c) $k = .1, \delta_1 = .1$

(d) $k = .1$

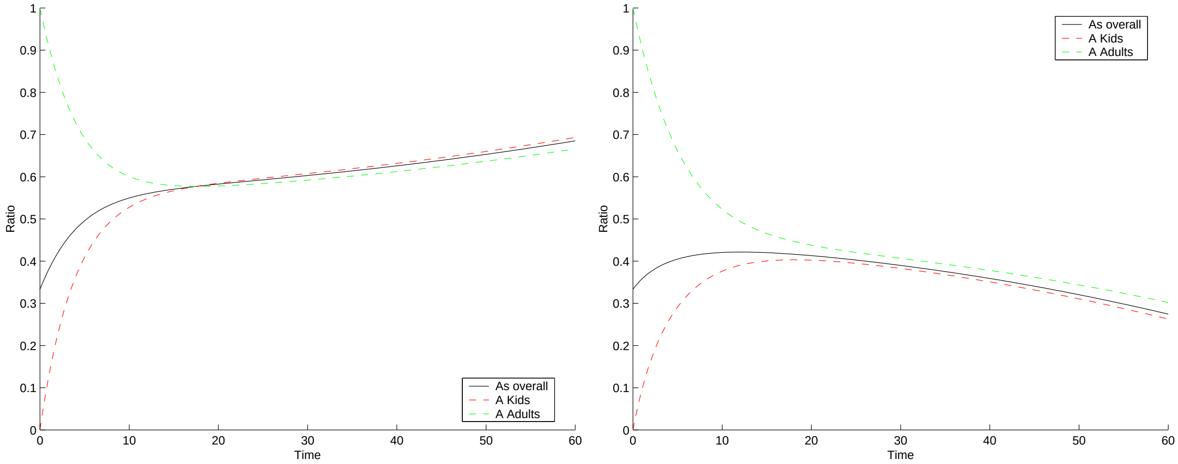
Figure 3: Short-Term Local Behavior with Age-Balanced Initial Population

long-term stage of development, and therefore, ultimately, the final outcome of the local model. Increasing k obviously drives the balance toward the adults' dominant grammar, while increasing δ_1, even in the presence of a high k-value, prefers the children's grammar. h does not generally have a strong effect on the short-term outcome of the model, although high h, by decreasing the frequency of children whose parents speak different grammars, does negate the effect of high δ_1 when k is also relatively large. In general, the default parameters give preference to the dominant grammar of the initial adult population, although the closer the grammar ratio is to .5, the more extreme the long-term consequences of small changes in the social parameters will be.



(a) Default Parameters

(b) $\delta_1 = .1$



(c) $k = .1, \delta_1 = .1$

(d) $k = .1$

Figure 4: Short-Term Local Behavior with Child-Dominated Initial Population

4.3 Global Model

4.3.1 Observations

The new population dynamics introduced by migration in the global model introduce no qualitatively different results on individual neighborhoods over a large time scale. That is, all neighborhoods still asymtotically approach a state in which they contain only A or B speakers. Though the global equilibrium state for a given initial neighborhood distribution is affected by the choice of various population dynamics parameters such as the population cap and the natural reproductive rate, individual neighborhoods always limit to domination of one grammar or the other.

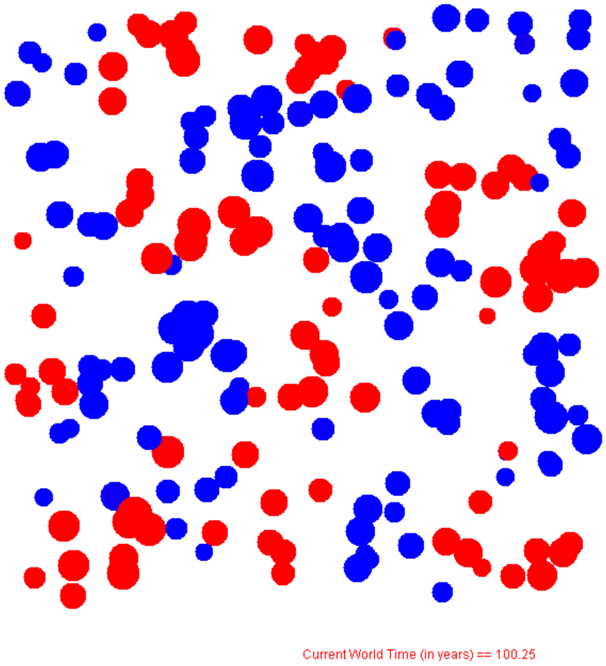


Figure 5: Long-Term Neighborhood Clustering with High Migration, $m = .03$, $t = 100$

The rate at which individual neighborhoods converge to **A** or **B** dominance is strongly affected by the migration rate parameter, m. For large values of m, neighborhoods converge quickly to one of the extremes. In these cases the global equilibrium state exibits signifcant clustering of neighborhoods sharing the same dominant grammar. Figure 5 shows clustering and early equilibration after 25 years (less than a quarter the usual equilibration time in the local model) with an abnormally high m value.

Psuedo-fixed points (as exhibited in figure 6) appear as result of the additional dynamics of the global model. The persistence of these points is strongly affected by m; higher migration rates lead to their early disappearance. The pseudo-equilibria tend to persist in regions with multiple nearby neighborhoods that have large populations of both grammars. That is they tend to lie between boundaries of regions defined by **A**-dominant clustering and **B**-dominant clustering.

In addition to the social and migration paramters, we may also manipulate the number of neighborhoods used in the global model. A threshold neighborhood density exists below which grammar clustering does not occur, simply because migration is so unlikely between distant neighborhoods. Above this threshold, we observe no significant qualitative change in global behavior.

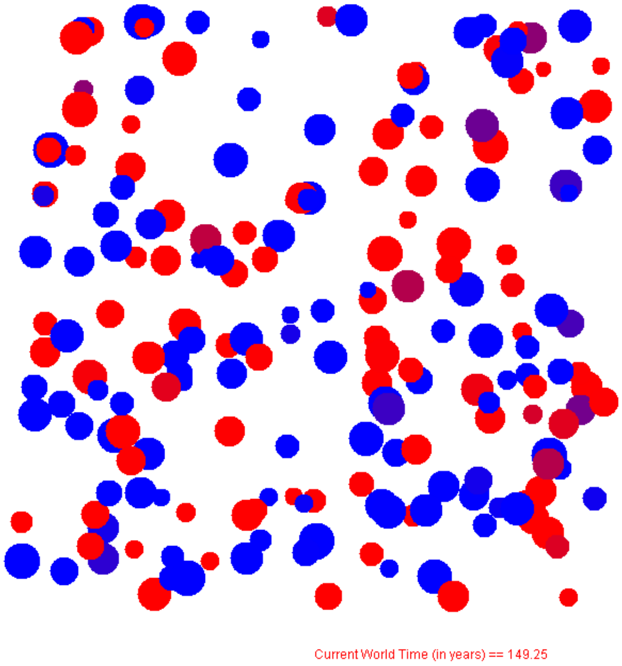


Figure 6: Boundary Pseudo-Equilibria Supported by Nearby Neighborhoods, $m = .0001$, $t = 149$

4.3.2 Analysis

The final population proportions of each neighborhood are not affected by emigration. Since migration decisions are based on expected reproductive rates, emigration gradually declines to zero as a neighborhood's population reaches its capacity. While migration can alter the destiny of early borderline populations, it only accelerates the local-level tendency toward the dominance of one grammar if that grammar gains a majority representation in the neighborhood.

The emigration parameter affects the final global outcome dramatically by spreading initial population proportion discrepancies throughout the immediate region of a neighborhood. This means that regions that begin with overall population proportions in favor of a single grammar will lead to the domination of not only those neighborhoods in the region that began with this bias, but all those neighborhoods that are sensitive to population pertubations of the magnitude presented by immigration.

The pseudo fixed points that appear along boundaries between opposite grammar dominated regions can be understood in terms of MacArthur's island meta-population theory. We can view the semi-stable grammar of the population as consisting of two species which are in danger of going extinct with some probability. The proximity of a mainland (large neighborhood) source population can help bolster the smaller population from losing either of its species. That is, the mainland acts as a buffer against **A** or **B** extinction.

Consider a dwindling subpopulation of grammar B within a neighborhood that has not yet reached capacity lying along such a boundary. This subpopulation may be bolstered by immigration from any of the neighborhoods with **B** preference along one part of the boundary. As the **B** subpopulation begins to dwindle to the point where B-speaking residents emigrate from the neighborhood, they free up room for new **B** immigrants coming from the "**B**-unfriendly" neighborhoods on the **A** side of the boundary and B-speakers coming from the at-capacity populations from the **B** side. Additionally, this neighborhood is made less desirable for both its **A** and **B** residents by the presence of one another, helping to keep the population turnover high enough to prevent the neighborhood from reaching its cap and equilibrating to domination by one grammar or the other. While this behavior is not sustainable in the very long term, we have observed pseudo-equilibria in our simulation that last several generations.

5 Sensitivity Summary

Initial behavior of the local model is unpredictable and based strongly on the initial population distribution and the social parameters. Within approximately one generation, the grammar proportions of the two age classes converge, and local behavior becomes very regular. Once both ratios fall on the same side of the unstable equilibrium at $\alpha_i = .5$, the population is guaranteed to tend toward the nearer extreme fixed point; that is, extinction of the minority population. Thus long-term behavior is very sensitive to initial conditions, though once the grammar ratio has equalized across the age classes, perturbations in the grammar ratios or social parameters will not affect the final outcome.

The global model yeilds qualitatively similar results regardless of initial population distribution constants, such as the maximum population capacity or the minimum initial distribution. The model is highly sensitive to changes in the migration factor, though this generally affects only the time scale of the global behavior; heavy migration does not result in any special equilibrium conditions. The other parameters of the model have no visually discernable effect on global behavior.

6 Strengths, Weaknesses, and Justifications

6.1 Strengths

- Symmetry of the two grammars is maintained throughout the design of the model. Without assuming any intrinsic differences between the two languages exist, the model yields asymmetrical results based purely on probabalistic interactions and stochastic initial conditions.

- The model differentiates between pre-breeding and adult populations, allowing it to address real-world issues of language aquisition and social pressure from different age groups such as peers or elders.

- Social pressures on grammar acquisition and the ecologically-inspired base model both make behavior highly contextual. Rather than having a fixed rate of imperfect learning, for example, children respond to the current state of the society in which they live.

- Global effects are incorporated, allowing the model to consider spatial developments in the evolution of the society. This manifests itself in the grammar clustering and other migration effects we observed.

6.2 Weaknesses

- Grammar aquisition is determined immediately upon birth rather than gradually throughout childhood. This is not entirely unrealistic for a model using only two age classes considering that the grammar aquisition among humans typically takes place during early childhood, during which time the social conditions influencing the child will not change significantly from the time when the child was born.

- The local model does not take into account spatial isolation of grammar groups in situations with skewed grammar ratios. Such self-isolationism can be observed in the real world, and may be an important part of language related demographics and the long term outcome of interacting populations that speak with different grammars. An earlier form of the local model incorporated a so-called “minority effect”, in which the mate-preference paramater was increased when one grammar was a minority. This simple solution produced no discernable local effects, so it was discarded. Regional properties of the grammar distribution are accounted for in the global model, however, and we observe the expected spatial isolation behavior.

- As the neighborhoods within the global model reach capacity as a whole, there is a reduction in the immigration and emigration. While a limitation, this is required for consistency with the notion of reproductive oppurtunity as a basis for emigration. To reduce the effect of this shortcoming, regional pertubations could be introduced that would maintain populations below their carrying capacity, or new neighborhoods could be allowed to be founded.

7 Conclusions

The emergence of regionally favored grammars in the global model illustrates the complexity of the interactions that can arise with only two competing grammars. Using only a probabalistic characterization of this relatively simple language dynamics problem, which locally always converges to extinction of one of the grammars, the model still gives interesting and intuitive results on the global scale. Our model suggests that the grammars reach an equilibrium state of geographical isolation over a time period on the order of three generations.

The existance of pseudo-fixed points as described in the results section are an encouraging parallel to linguistically rich locations that often exist near the boundary of two distinct cultural epicenters. The appearence of these points suggests that the model may bear significant predictive properties. Additionally, the longterm regionality observed is consistent with the regional domination of many languages. This effect of spatial segregation based on language choice over the time frame modeled is realistic and further supports the models validity.

The results obtained here may be extended with only minor modification to explore more long term interactions or other ideas. One option, discussed above, is the introduction of destabilizing effects on neighborhoods at population caps. Other possible extensions of this model include expanding the number of grammars or age groups, or expanding the scope to a super-global aggregation of models similar to our current global model.